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Study on Dynamics of Hypocycloid RV Reducer

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Abstract: The hypocycloid RV reducer is a new type of rotor vector (RV) reducer based on the principle of internal cycloidal planetary transmission. The reducer has a compact and hollow structure. First of all, the component composition, special structure and transmission principle are analyzed. Then, the three-dimension dynamic model for the hypocycloid RV reducer was simulated. Through reasonable assumptions, the dynamic environment was defined, the dynamic model was simplified, and the kinetic equations were listed. The torsional stiffness of the shaft component is calculated, and the meshing rigidity of the cycloidal gear and the needle tooth is calculated. Finally, by using MATLAB software to solve the dynamic equations, combined with the actual parameters of the hypocycloid RV reducer, through the solving equation to obtain the natural frequency of the system, which laid the foundation for the design and research of the following hypocycloid RV reducer.

Keywords: rotor vector(RV) reducer; dynamic model; natural frequency

1 Introduction

Rotor vector (RV) reducer is a kind of two-stage precision reducer. It has the advantages of small size, light weight, large transmission ratio range, long life and high precision. It is widely used in precision transmission fields such as industrial robots, high-precision machine tools, medical equipment, and aerospace. During the operation of the equipment, there is a vibration of the whole system of the equipment caused by the operation of the reducer. This condition will not only affect the working accuracy of the equipment but also reduce the service life of the reducer. Therefore, it is necessary to study the dynamics of the RV reducer. In this paper, a certain cycloidal RV reducer is taken as the research object, and the corresponding dynamic model of the whole machine is established. The kinetic formula is given and the natural frequency is obtained by solving it. This provides a reference for the parameter design of the reducer and future research.

Regarding such topics, many domestic and foreign scholars have studied it. Carlo[1] studied the structural motion characteristics of the cycloidal transmission part and explained the kinematics principle; Kim[2] calculated the contact stress of the planetary transmission and the torsional stiffness of the mechanism; Wang[3] established a 3D simulation model of 2 K-V two-stage reducer, based on which the dynamic model was established and the dynamic performance was studied; Shan[4] established a nonlinear dynamic model of the two-stage cycloidal planetary transmission, and analyzed the nonlinear dynamic characteristics; Li[5-6] through the three-dimensional simulation software, the RV reducer was powered Simulation analysis of learning characteristics; Liu[7] added the tangential translational freedom of the cycloidal wheel and crankshaft revolutions in the established model, and

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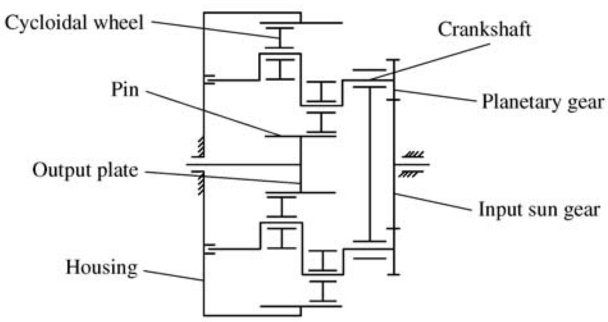
analyzed the natural frequency and sensitivity; Yan[8] Considering the degree of freedom of each component in the RV reducer, a translation-torsion dynamic model is established to solve the natural frequency of the RV reducer.

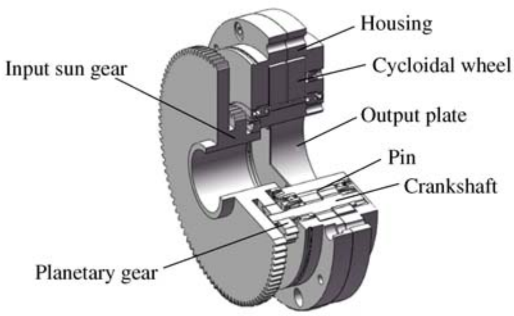
The above literatures are all aimed at the study of the dynamic characteristics of the external cycloidal RV reducer, but the whole machine dynamics model of the hypocycloid RV reducer is not involved, and the related dynamics research is relatively rare. In this paper, the dynamics model of the hypocycloid RV reducer is used as a kinetic study.

2 Structure and transmission principle

The hypocycloid RV reducer is a two-stage precision transmission system consisting of a first-stage involute planetary gear structure and a second-stage cycloidal structure, as shown in Figures 1. The first stage is composed of the input sun gear and the planetary gear. The sun gear and the input gear are integrally connected, the input gear rotates, and the sun gear rotates at the same time so that the planetary gear rotates in reverse. The second stage consists of two symmetrically arranged crankshafts, two upper and lower arranged cycloidal wheels, and circumferentially arranged pins and output plate. One end of the crankshaft is connected to the planetary gear, and the other end is matched to the shaft hole of the housing, and the middle portion is matched with the cycloidal wheel. The planetary gear is connected with the crankshaft spline, and the crankshaft rotates with the planetary gear. The cycloidal wheel is revolved by the rotation motion of the crankshaft, and the output plate is rotated at the same time. Finally, the output plate will be self-rotating feedback output.



a) Institution diagram of hypocycloid RV reducer



b) Three-dimensional model of hypocycloid RV reducer

Figure 1 Structure of hypocycloid RV reducer

3 Establishment of dynamics model

Since the hypocycloid RV reducer is composed of two-stage transmission, its mechanical motion is more complicated, and there are many factors affecting its vibration characteristics. It is difficult that establish a dynamic model when influencing factors should be thoroughly examined. Therefore, some assumptions must be made to its dynamic model in the early stage. The specific assumptions are as follows:

1) The centroid of each component coincides with its geometric center;

2) The gear meshing stiffness and bearing stiffness are constant, ignoring the nonlinear change of its

stiffness;

3) Neglect the influence of the friction and damping of the contact surface;

4) The force between the cycloidal wheel and the pins is equivalently replaced by the concentrated force;

5) The sun gear speed does not change;

6) Neglect the effects of internal lubrication.

Based on the above-mentioned assumptions, the following simplified ideas can be obtained for the structure of the hypocycloid RV reducer:

1) The sun gear and the planetary gear meshing transmission are equivalently replaced by a spring connection;

2) The planetary gear is connected to the crankshaft by a spline. Since the spline connection structure is firm, it is regarded as a concentrated mass in the direction of torsional freedom, and is replaced by a spring connection;

3) The crankshaft is connected to the cycloidal wheel through the slewing bearing and it is connected to the housing through the support bearing. Because the bearing has large elasticity and small mass, the bearing is equivalently replaced by a spring connection;

4) The cycloidal wheel meshes with the pins. Since the number of pins is large, the meshing condition is complicated, so it is simplified to be equivalent to two concentrated masses, and the meshing contact portion is replaced by a spring connection.

Based on the above-mentioned analysis, the simplified model includes the sun gear, the planetary gear, the crankshaft, the cycloidal wheel and the output plate. Establish a system dynamics model for it, as shown in Figure 2.

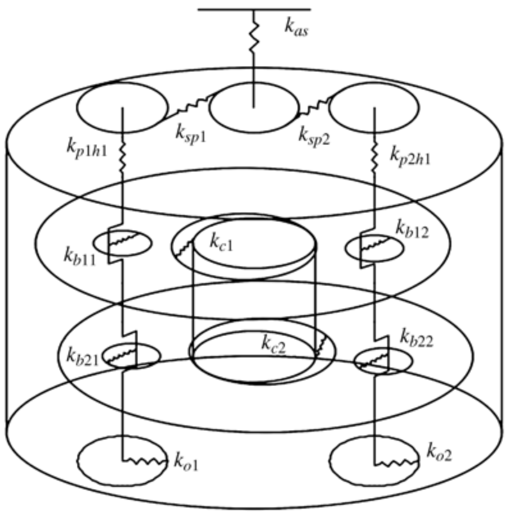


Figure 2 Dynamic model for hypocycloid RV reducer

The specific dynamic structure features are as follows:

1) Revolving angle of the sun gear θ_s, revolving angle of the planetary gear θ_{pi}, revolving angle of the cycloidal wheel θ_{ci};

2) Evolution center displacement of the cycloidal wheel X_{ci};

3) Revolving angle of the crankshaft θ_{hi}.

4) Revolving angle of the input gear θ_a, revolving angle of the output plate θ_o.

Finally, the simplified kinetic model has a total of 11 degrees of freedom, with both line displacements and angular displacements. In order to facilitate the calculation, when determining the generalized coordinates, all the dimensions are converted into line displacements, and 11 generalized coordinates are obtained:

The micro-displacement of the sun gear along the meshing action line is shown as

$$x_s = r_{bs} \cdot \theta_s$$

Where, r_{bs} is the base radius of the sun gear.

The micro-displacement of the planetary gear along the meshing action line is shown as

$$x_{pi} = r_{bpi} \cdot \theta_{pi}$$

Where, r_{bpi} is the base radius of the planetary gear.

The tangential micro-displacement of the eccentric cam center of the crankshaft is in the following

$$x_{hi} = a \cdot \theta_{hi}$$

Where, a is the eccentricity.

The tangential micro-displacement of the cycloidal wheel is shown as

$$x_{ci} = R_h \cdot \theta_{ci}$$

Where, R_h is the circle radius of the crankshaft.

The tangential micro-displacement of the input gear is shown as

$$x_a = r_{bs} \cdot \theta_a$$

The tangential micro-displacement of the output plate is shown as

$$x_o = R_h \cdot \theta_o$$

The micro-displacement of the center of the cycloidal wheel is X_{ci}, and the value of i is 1 or 2.

4 Kinetic equations

The RV reducer is a typical discrete vibration system. After selecting the generalized coordinates, the Lagrange equation can be used to derive the equation of motion of the system:

$$m_a \ddot{x}_a + k_{as}x_a - k_{as}x_s = p_I$$

$$m_s \ddot{x}_s + k_{as}x_a - k_{as}x_s + \sum_{i=1}^{2} k_{spi}(x_s - x_{pi}) = 0$$

$$m_{pi} \ddot{x}_{pi} - k_{spi}x_s + k_{spi}x_{pi} - k_{pihi}x_{hi} + k_{pihi}x_{pi} = 0$$

$$m_{hi} \ddot{x}_{hi} - k_{pihi}[(-1)^i x_{pi} - x_{hi}] + k_{bi1}[X_{c1} + (-1)^{i+1}x_{c1} - x_{h1} \cdot a - (-1)^i x_o] \cdot a - k_{bi2}[X_{c2} + (-1)^{i+1}(x_{c2} - x_{h1} \cdot a - x_o)] \cdot a + k_{oi}(x_{hi} - x_o) = 0$$

$$m_{ci} \ddot{x}_{ci} + k_{ci}x_{ci} - (-1)^i k_{b1i}X_{ci} + k_{b1i}x_{ci} - k_{b1i}\, ax_{h1} - k_{b1i}x_o + (-1)^i k_{b2i}X_{c2} + k_{b2i}x_{ci} - (-1)^i k_{b2i}\, ax_{h2} - k_{b2i}x_o = 0$$

$$M_{ci} \ddot{X}_{ci} + k_{b1i}X_{ci} - (-1)^i (k_{b1i}X_{ci} - k_{b1i}\, ax_{h1} - k_{b1i}x_o - k_{b2i}x_{ci} + k_{b2i}\, ax_{h2} + k_{b2i}x_o) + k_{bi2}X_{ci} = 0$$

$$m_o \ddot{x}_o - \sum_{i=1}^{2} k_{hi} X_{pi} + \sum_{i=1}^{2} (k_{b11} - k_{b12}) a x_{hi} - \sum_{i=1}^{2} k_{hi} X_{pi} - \sum_{i=1}^{2} \sum_{j=1}^{2} k_{bij} x_{ci} - \sum_{i=1}^{2} k_{oi} (x_{hi} - x_o) + (k_{b21} - k_{b11}) X_{c1} +$$
$$(k_{b12} - k_{b22}) X_{c2} + \left(\sum_{i=1}^{2} \sum_{j=1}^{2} k_{bij} + \sum_{i=1}^{2} k_{ci} \right) x_o = - p_o$$

Where: $m_a = l_a / r_{bs}^2$, m_a is the equivalent mass of the input gear, l_a is the moment of inertia of the input gear, r_{bs} is the base radius of the sun gear; $m_s = l_s / r_{bs}^2$, m_s is the equivalent mass of the sun gear, l_s is the moment of inertia of the sun gear; $m_{pi} = l_{pi} / r_{bpi}^2$, m_{pi} is the equivalent mass of the planetary gear, l_{pi} is the moment of inertia of the planetary gear, r_{bpi} is the base radius of the planetary gear; $m_{hi} = l_{hi} / a^2$, m_{ci} is the equivalent mass of the crankshaft, l_{hi} is the moment of inertia of the crankshaft, a is the eccentricity; $m_{ci} = l_{ci} / R_h^2$, m_{hi} is the equivalent mass of the cycloidal wheel, l_{ci} is the moment of inertia of the cycloidal wheel, R_h is the circle radius of the crankshaft; $m_{oi} = l_{oi} / R_h^2$, m_{oi} is the equivalent mass of the output plate, l_{oi} is the moment of inertia of the output plate; M_{ci} is the quality of the cycloidal wheel; α is the pressure angle of the involute gear; k_{spi} is the meshing stiffness of the planetary gear; k_{ci} is the meshing stiffness of a cycloidal wheel; k_{as} is the torsional stiffness of the input gear; k_{pihi} is the torsional stiffness of the crankshaft near one end of the planetary gear; k_{hi} is the bending stiffness at the planetary gear mounted on the crankshaft; k_{bij} is the bending stiffness at the cycloidal wheel mounted on the crankshaft; k_{oi} is the bending stiffness of the end of the crankshaft; Subscript a is the stiffness of the crankshaft near the end of the planetary gear; Subscript b is the stiffness of the crankshaft near the end of the cycloidal wheel; P_I is the generalized force at the input; P_o is the generalized force at the output; $i = 1, 2$; $j = 1, 2$.

Convert the above equation into a general form of the mechanical equation represented by the following matrix

$$[\boldsymbol{M}]\{\ddot{\boldsymbol{x}}\} + [\boldsymbol{K}]\{\boldsymbol{x}\} = \{\boldsymbol{f}\}$$

Where, $[\boldsymbol{K}]$ is the in the equation is a stiffness matrix that changes with time. The equation is a variable coefficient differential equation.

5 Solution of kinetic equations

5.1 Torsional stiffness of the shaft

According to the theory of material mechanics, the torsional stiffness of a uniform cylinder is

$$k = GI_p / l$$

Where: G is the elastic shear modulus; I_P is the polar moment of inertia; l is the length of the shaft.

5.2 Intermeshing stiffness of involute gear

According to the material mechanics method of the involute gear meshing stiffness, the formula for calculating the meshing stiffness of a single pair of meshing teeth is shown as

$$k = \frac{C_M C_K C_B}{q} \left\{ \frac{0.75}{2\pi} [Z_1(\tan\alpha_{a1} - \tan\alpha) + Z_1(\tan\alpha_{a1} - \tan\alpha)] + 0.25 \right\}$$

Where: C_M is the theoretical correction value, $C_M = 1$; C_B is the basic profile coefficient, $C_B = [1+0.5(1.2+h_{fp}/$

$m)]\times[1-0.02(20°-\alpha)]$; C_K is the wheel slab structure coefficient; q is the flexibility of the unit tooth width; Z_1 is the number of gear teeth; Z_2 is the number of large gear teeth; α is the meshing angle of the gear; α_{a1}, α_{a2} are the pressure angle of the tip crown.

5.3 Meshing stiffness of cycloidal wheel and pins

As shown in Figure 3, the cycloidal wheel rotates clockwise, and the lower cycloidal wheel of the X-axis does not generate contact force with the pins. Above the X-axis, the cycloidal wheel is in meshing contact with the pins to generate a contact force. Under the action of the moment T, the cycloidal wheel is in contact with the ith pin in the direction of the common normal of the contact point, and its deformation displacement s_i is as follow

$$s_i = \Delta\beta \cdot l_i$$

Where: l_i is the vertical distance between the rotation center O_a of the cycloidal wheel and the normal direction of the ith pin force point; $\Delta\beta$ is the angle at which the cycloidal wheel rotates around the center of rotation O_a.

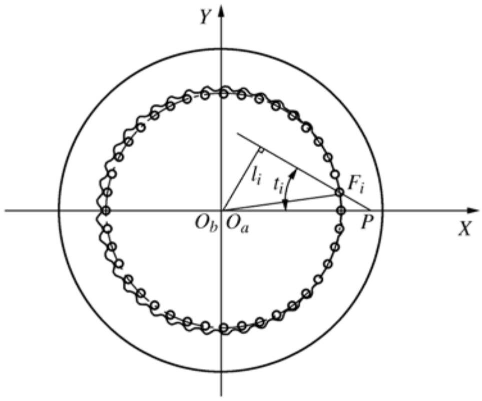


Figure 3 Engaging force of cycloid structure

Let the force of the ith pin acting on the cycloidal wheel be F_i, when $l_i = r_a$, we can get

$$s_{amx} = \Delta\beta \cdot r_a$$

Where, r_a is the pitch circle radius of the cycloidal wheel.

The system transmits torque from two cycloidal wheels. Due to the error in the production process and the characteristics of the structure, the torque transmitted to the two cycloidal wheels is different, so take $T = 0.55\ T_a$, that is

$$F_{max} = \frac{4T}{z_a r_a} = \frac{2.2T_a}{z_a r_a}$$

Under the action of F_{max}, the maximum contact deformation s_{max} of the contact point of the pair of teeth with the largest force in the common normal line direction is

$$s_{max} = \frac{2(1-\mu^2)}{E}\frac{F_{max}}{\pi b}\left(\frac{2}{3} + \ln\frac{16r_z\,|\,\rho\,|}{c^2}\right)$$

Where: μ is the poisson's ratio of the cycloidal wheel and the pin; E is the elastic modulus of the cycloidal wheel and the pin; r_z is the radius of the pin; b is the width of the cycloidal wheel; ρ is the tooth profile radius of curvature of the cycloidal wheel; $\rho = r_z r_c / (r_z + r_c)$, r_c is the replace arc radius.

Finally, the torsional meshing stiffness of the cycloid at time t is

$$k_a = \frac{T_a}{\Delta\beta}$$

5.4 Results and analysis

The parameters of the internal cycloid RV reducer are: sun gear teeth $z_s = 50$, planetary gear teeth $z_p = 20$, modulus $m = 1$ mm, tooth angle $\alpha = 20°$, cycloidal wheel tooth number $z_a = 41$, pin number $z_b = 40$, eccentricity $a = 1$ mm, needle tooth radius $r_z = 4$ mm, pin tooth distribution circle radius $R_z = 100$ mm, speed ratio $i = 17$, output speed $n = 30$ r/min, rated load $T_a = 250$ N · m.

Considering that the traditional solution method is inefficient and inaccurate, in order to calculate the natural frequency and modal shape of the system more accurately, the numerical calculation is performed by MATLAB software. Obtain the moment of inertia of each component from the 3D model, edit the MATLAB program, substitute the values, and find the natural frequency and mode shape of the system. Due to space limitations, only the first three natural frequencies (132 Hz, 913 Hz, 1 452 Hz) and modal shape data of the results are given, as shown in Table 1.

Table 1 Natural frequencies and mode shapes

Freedom	Modal shape		
x_a	0.911	0.000	-1.200
x_x	0.911	0.000	-0.974
x_{p1}	1.000	-1.000	0.258
x_{p2}	1.000	1.000	0.258
x_{c1}	-0.015	-0.118	-0.387
x_{c2}	-0.015	0.118	-0.387
X_{c1}	-0.250	-0.236	0.121
X_{c2}	-0.250	0.236	0.121
x_{h1}	0.235	-0.496	0.273
x_{h2}	0.235	0.496	0.273
x_o	-0.319	0.000	0.012

6 Conclusions

Aimat taking the hypocycloid RV reducer, combining with the actual physical object, the simulation of the 3D dynamic model for whole machine is carried out, and the dynamic equations are listed. By using MATLAB software to solve its natural frequency and modal shape, comparing with the actual situation to meet the simulation requirements, laying the foundation for the research and structural optimization of the hypocycloid RV reducer.

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