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Study on Prediction Method of Structural Three-parameter Power Function P-S-N Curve

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Abstract: This paper applies weighted least square method to estimate the three-parameter power function equation of the fatigue life curve, and uses comprehensive fatigue life coefficient to correct the equation, and at the same time combines probability statistics method to bring out the prediction method of structure's three-parameter power function P-S-N curve, finally applies the prediction method to a ship's frame-type elevate, based on the fatigue test data of it's material-5A06 aluminium alloy, to obtain it's structure's three-parameter power function P-S-N curve. Compared with the conventional least square method, the presented method can give a rational estimation due to it's a best linear unbiased estimator, and this method has stronger practicality in engineering, and can be used in many structures of different materials, for achieving their three-parameter power function P-S-N curve.

Keywords: P-S-N curve; comprehensive fatigue life coefficient; three-parameter power function; fatigue life

1 Introduction

Structure's fatigue curve based on reliability is very important for analyzing and designing structural reliability[1-2], At present, the study of the fatigue curve based on reliability is wide, Ref[3] applied the Box-Cox variable transformation fit test to estimate the three-parameter P-S-N curve, Ref[4] applied double-weighted least square method to estimate P-S-N curve, Ref[5] applied Shapiro test method to estimate the P-S-N curve of steel welding joint under several certain reliabilities, Ref[6] applied the statistical method of augmented sample to estimate the P-S-N curve of D32 steel, Ref[7] applied the method of the extended small sample of the time-varying random model to estimate P-S-N curve, Ref[8] applied the equivalence principle of the stress amplitude variation and the life cycle variation to estimate high-reliability P-S-N curve, Ref[9] applied the heteroscedasticity regression theory to estimate a small sample P-S-N curve. All the methods above for studying the P-S-N curve are used for the materials, but for the structure, we usually apply the method of comprehensive coefficient correction to estimate structure's P-S-N curve under several certain reliabilities.

2 Three-parameter fatigue curve model

The formula of the three-parameter fatigue curve is shown below

$$\lg N = \lg C - m \lg(S - A) \tag{1}$$

Where: m and C are material constant; S is maximum stress of samples; N is fatigue life of samples; A is the maximum of the theoretical fatigue limit and it's value range is $[0, S_{min}]$, where S_{min} is the minimum of all the

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stress level.

Hypothesis $x=\lg N, y=\lg(S-A), a=\lg C, b=-m$, the Formula (1) can be written as

$$x = a + by \tag{2}$$

Using the correlation coefficient ρ_{xy} to express the linear relationship between x and y, and the expression of the correlation coefficient is shown as

$$\rho_{xy} = \frac{L_{xy}}{\sqrt{L_{yy} \times L_{xx}}} \tag{3}$$

$$L_{xy} = \sum_{j=1}^{n} x_j y_j - \frac{1}{n}\left(\sum_{j=1}^{n} y_j\right)\left(\sum_{j=1}^{n} x_j\right) \tag{4}$$

$$L_{xx} = \sum_{j=1}^{n} x_j^2 - \frac{1}{n}\left(\sum_{j=1}^{n} x_j\right)^2 \tag{5}$$

$$L_{yy} = \sum_{j=1}^{n} y_j^2 - \frac{1}{n}\left(\sum_{j=1}^{n} y_j\right)^2 \tag{6}$$

$$x_j = \overline{\lg N^j} = \frac{1}{k}\sum_{i=1}^{k} \lg N_i^j \tag{7}$$

$$y_j = \lg(S_j - A) \tag{8}$$

Where: j is stress level $(j=1\cdots n)$; N_i^j is the fatigue life of sample i under the stress level of $j(i=1\cdots k)$; S_j is the stress of level j; x_j is the stress's mean value of logarithmic fatigue life of level j.

Adopting the iterative method of the literature[11-12] to the Formula (3), we can calculate the value of A, and then we can calculate the each value of a and b based on the weighted least square method of the literature [13]. Substituting a、b and A into the Formula (1), we can get the fatigue curve equation of the three-parameter power function of the material as

$$\lg N = a + b\lg(S - A) \tag{9}$$

3 Power function model of three-parameter structure based on reliability

This paper will focus on the influence of fatigue notch coefficient K_f、size coefficiente and surface sensitivity coefficient η on the fatigue life of structures, and define the function of the comprehensive correction coefficient by the three factors, so the function can be written as[14]

$$\lambda = \frac{K_f}{e} + \frac{1}{\eta} - 1 \tag{10}$$

Fatigue life of structures under the same stress is often less than that of materials, after considering the comprehensive correction on the basis of Formula (1), the structures' fatigue life curve equation of the three-parameter power function is shown below:

$$\lg N = \lg C - m\lg[(S - A)\lambda] \tag{11}$$

When adopting group method to test the fatigue life, N stress levels $[S_1 \quad S_2 \quad \cdots \quad S_n]$ are selected, and k stress fatigue tests are carried out at each stress level to obtain n groups of the logarithm fatigue life test data $[\lg N_1^1 \quad \lg N_2^1 \quad \cdots \quad \lg N_k^1]$、$[\lg N_1^2 \quad \lg N_2^2 \quad \cdots \quad \lg N_k^2]$、…、$[\lg N_1^n \quad \lg N_2^n \quad \cdots \quad \lg N_k^n]$, according to the characteristics of logarithmic fatigue life obeying normal distribution[15], we can obtain n groups of the mean and variance of logarithm fatigue life $(x_1 \quad \sigma_1^2)$, $(x_2 \quad \sigma_2^2)$,…, $(x_n \quad \sigma_n^2)$, then put the data $(S_1 \quad x_1)$、

$(S_2 \quad x_2) \cdots (S_n \quad x_n)$ into the Formula(3) ~ Formula (8), and the value of parameter A can be obtained by iterative method when the structure reliability is 0.5, we see it as $A_{p=0.5}$, then, according to the weighted least square method of literature[13], we can get the value of a and b when the reliability of structure is 0.5, respectively $\lg C_{p=0.5}$、$-m_{p=0.5}$. Substituting a、b and A into the Formula (10), we can get the stress life curve of a structure with a probability of 0.5, its formula is shown as

$$\lg N_{p=0.5} = \lg C_{p=0.5} - m_{p=0.5} \lg[(S - A_{p=0.5})\lambda] \tag{12}$$

Where: $N_{p=0.5}$ is fatigue life at the reliability of 0.5; $C_{p=0.5}$ and $m_{p=0.5}$ are the regression coefficients for the reliability of 0.5; $A_{p=0.5}$ is the theoretical stress fatigue limit at the reliability of 0.5.

According to the characteristics of the normal distribution[16], the logarithmic fatigue life of p_1 per group of test data is expressed as follows:

$$x_{j,p_1} = x_j + \alpha_{p_1} \sigma_j \tag{13}$$

Where: x_{j,p_1} is the logarithm fatigue life of group $j(j=1,2\cdots n)$ when reliability is p_1; x_j is the logarithm fatigue life of group $j(j=1,2\cdots n)$ when reliability is 0.5; α_{p_1} is the lower-side digits of the standard normal distribution when reliability is p_1; σ_j is the standard deviation of logarithmic fatigue life of group $j(j=1,2\cdots n)$.

According to the formula (12), we can get the data groups when the reliability is 0.5, such as $(S_1 \quad x_{1,p1})$、$(S_2 \quad x_{2,p1})$、$\cdots$、$(S_n \quad x_{n,p1})$, according to the solving process of the structure's stress life curve when the reliability is 0.5, the above data groups are processed to obtain the value of the parameter A when the reliability is p_1, that is $A_{p=p_1}$, and the values of regression coefficients a、b are $\lg C_{p=p1}$、$-m_{p=p1}$, respectively, substituting them into the Formula (10), the stress life curve of the structure can be obtained when the reliability is p_1, and its expression is shown as

$$\lg N_{p=p1} = \lg C_{p=p1} - m_{p=p1} \lg[(S - A_{p=p1})\lambda] \tag{14}$$

Where: $N_{p=p_1}$ is the fatigue life when the reliability is p_1; $C_{p=p_1}$、$m_{p=p_1}$ are the regression coefficients when the reliability is p_1; $A_{p=p_1}$ is the theoretical stress fatigue limit when the reliability is p_1.

Substituting the Formula (12) and Formula (14) into the Formula (13), we can get the expression of standard deviation of logarithmic fatigue life under arbitrary stress level, and its expression is shown below

$$\sigma = \left(\lg \frac{C_{p=p1}}{C_{p=0.5}} - m_{p=p1} \lg[(S - A_{p=p1})\lambda] + m_{p=0.5} \lg[(S - A_{p=0.5})\lambda] \right) / \alpha_{p1} \tag{15}$$

Using the relationship between variables in the Formula(13), the fatigue life curve equation of structure three-parameter power function based on reliability can be shown as

$$\lg N_p = \lg C_{p=0.5} - m_{p=0.5} \lg[(S - A_{p=0.5})\lambda] + \frac{\alpha_p}{\alpha_{p1}} \{ \lg \frac{C_{p=p1}}{C_{p=0.5}} - m_{p=p1} \lg[(S - A_{p=p1})\lambda] + m_{p=0.5} \lg[(S - A_{p=0.5})\lambda] \} \tag{16}$$

Where: N_p is the fatigue life of structures under arbitrary reliability; α_p is the lower-side digits of standard normal distribution.

4 Application instance

In this paper, we establish the fatigue life curve equation of a frame-type ship lift platform based on reliability by using 36 specimens of fatigue test on 5A06 aluminum specimens. Figure 1 is the fatigue specimen of aluminum alloy, the sample is divided into 6 groups, and two groups remain as standby. Fatigue test is carried on the LFV-L-12KN fatigue testing machine (Figure 2), the test environment is standard atmospheric pressure, the test temperature is room temperature, the stress ratio is 0.1, the test load waveform is the sine wave, the test frequency is 25 Hz. Sample installation as shown in Figure 3, the fatigue life values at all levels of the specimen are listed in Table 1.

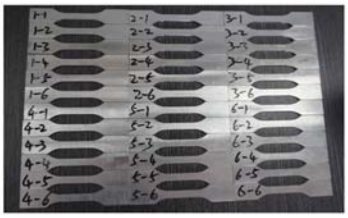

Figure 1 Fatigue test samples

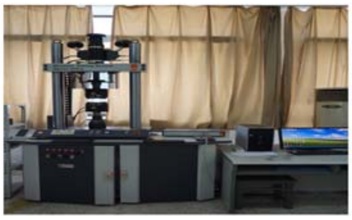

Figure 2 Fatigue testing machine

Figure 3 The installation and failure of fatigue samples

Table 1 Fatigue life data of the 5A06 aluminium alloy plate

Maximum stress/MPa	Test specimen	Log fatigue life	Mean value	Variance
		4.577 26, 4.724 66		
225	6	4.589 09, 4.720 41		
		4.860 66, 4.754 02	4.704 35	0.001 6
		4.829 98, 5.042 66		
201	6	4.779 4, 4.576 49		
		4.789 66, 4.776 62	4.799 14	0.003 1
		5.142 89, 5.116 87		
180	6	5.553 22, 5.372 38		
		5.146 63, 5.533 13	5.310 85	0.005 7
		5.524 19, 5.379 08		
167.5	6	5.340 21, 5.795 15		
		5.558 9, 5.842 85	5.573 4	0.006

Substituting the fatigue test data into the Formula (3) ~ Formula (8), the fatigue life curve of the material with the reliability 0.5 is obtained by using the weighted lest squares method of Ref[13], and its expression is shown as

$$\lg N_{p=0.5} = 7.411\ 9 - 1.457\ 9\lg[(S - 149)] \tag{17}$$

As for the ship frame lifting platform (Figure 4), according to the calculation method of the Ref[18] and combining the characteristics of the lifting platform itself, we can calculate the value of the fatigue notch coefficient K_f、size coefficiente and surface sensitivity coefficient η, that is $K_f = 1$、$e = 0.96$ and $\eta = 0.74$.

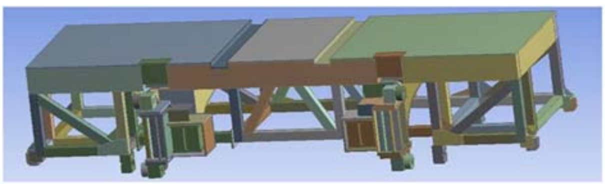

Figure 4 The 3D model of a ship frame lifting platform

Substituting K_f、e and η into the Formula (9), we can calculate the value of the comprehensive correction coefficient λ, that is $\lambda = 1.4$. Using λ to correct the Formula (17), the S-N curve of the lifting platform structure can be obtained when the reliability is 0.5, and its expression is shown as

$$\lg N_{p=0.5} = 7.411\ 9 - 1.457\ 9\lg[(S - 149) \times 1.4] \tag{18}$$

When $\alpha_{p1} = -1$, the reliability is 0.841 3. Given the mean and variance of the logarithm fatigue life and using the Formula (3) ~ Formula (8), we can get the S-N curve of the lifting platform structure when the reliability is 0.841 3, and its equation can be expressed as

$$\lg N_{p=0.841\,3} = 6.737\,9 - 1.161\,8\lg[(S - 153) \times 1.4] \tag{19}$$

We can get the relationship between the variance and the stress from Formula (14), and the relationship can be shown below

$$\sigma = \lg N_{p=0.5} - \lg N_{p=0.8413} = 0.674 + \lg\left\{\frac{[(S - 153) \times 1.4]^{1.161\,8}}{[(S - 149) \times 1.4]^{1.457\,9}}\right\} \tag{20}$$

The distribution graph of the logarithmic fatigue life probability density function of the ship lift platform structure can be obtained by Formula (18) and Formula (20), as shown in Figure 5.

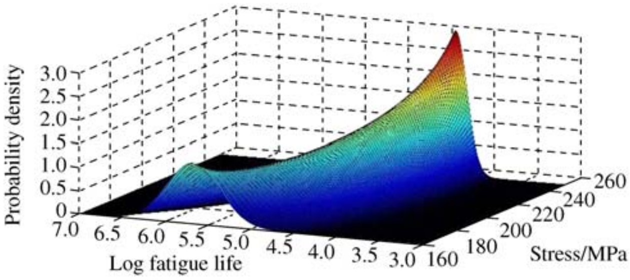

Figure 5 The probability density of the structure's logarithmic fatigue life

Substituting the Formula (18) and Formula (20) into Formula (16), we can obtained the logarithmic fatigue life curve equation of the ship lifting platform based on reliability, and its expression is shown as

$$\lg N_p = 7.411\,9 - 1.457\,9\lg((S - 149) \times 1.4) + \alpha_p\left[0.674 + \lg\left(\frac{((S - 153) \times 1.4)^{1.161\,8}}{((S - 149) \times 1.4)^{1.457\,9}}\right)\right] \tag{21}$$

When α_p takes 0、−1、−1.645 and −3.326, separately, the structural fatigue life can be obtained as the reliability takes 0.5、0.841 3、0.95 and 0.99, and its curve is shown in Figure 6.

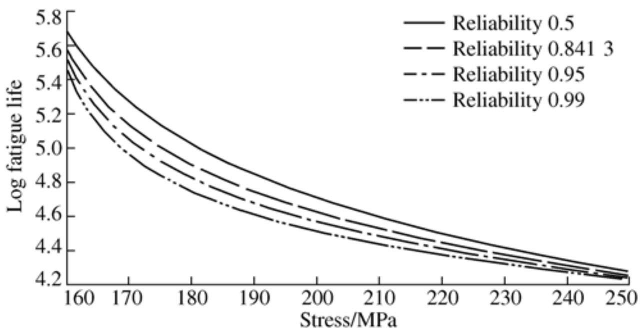

Figure 6 The structural fatigue life curves under four reliabilities

It is shown from Figure 6 that, under the same stress condition, the higher the reliability is, the smaller the logarithmic fatigue life and the variance of the logarithm fatigue life increases with the decrease of the stress.

5 Conclusions

This paper presents a three-parameter power function P-S-N curve prediction model, which is based on a small amount of fatigue test sample data and takes fully into account the related factors affecting the fatigue life of structures, and adopts the method of comprehensive coefficient correction. From the example, we can see that the logarithmic fatigue life of the structure obeys the normal distribution under the assumption that the logarithmic fatigue life of the specimen obeys the normal distribution. The prediction model can be applied to the fatigue life prediction of different materials under arbitrary reliability, and it has higher engineering practicability and improves the efficiency of solving the P-S-N curve of structure.

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