

Adaptive Bearing Fault Diagnosis based on Wavelet Packet Decomposition and LMD Permutation Entropy

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Abstract: Bearing fault signal is nonlinear and non-stationary, therefore proposed a fault feature extraction method based on wavelet packet decomposition (WPD) and local mean decomposition (LMD) permutation entropy, which is based on the support vector machine (SVM) as the feature vector pattern recognition device. Firstly, the wavelet packet analysis method is used to denoise the original vibration signal, and the frequency band division and signal reconstruction are carried out according to the characteristic frequency. Then the decomposition of the reconstructed signal is decomposed into a number of product functions (PE) by the local mean decomposition (LMD), and the permutation entropy of the PF component which contains the main fault information is calculated to realize the feature quantization of the PF component. Finally, the entropy feature vector input multi-classification SVM, which is used to determine the type of fault and fault degree of bearing. The experimental results show that the recognition rate of rolling bearing fault diagnosis is 95%. Comparing with other methods, the present this method can effectively extract the features of bearing fault and has a higher recognition accuracy.

Keywords: fault diagnosis; wavelet packet decomposition (WPD); local mean decomposition (LMD); permutation entropy; support vector machine(SVM)

1 Introduction

In the early stage of bearing damage, the typical non-stationary, modulation and weak characteristics are shown, and a large number of shock and noise are included in the vibration signal of bearing based on vibration

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acceleration sensor. Therefore, it is very difficult to extract the weak signal characteristics of the bearing running state from the strong background noise. The traditional time domain and frequency domain methods have their own limitations for the non-stationary time varying signal of the bearing fault.

Non stationary signal adaptive decomposition method has been widely used in the field of fault diagnosis. The local mean decomposition (LMD) method is a new signal processing method proposed based on the empirical mode decomposition (EMD) method by Smith Jonathan et al. Due to its self-adaptive time-frequency analysis, it has been widely used in the analysis of nonlinear and non-stationary signals. Ma et al.^[1] proposed a rolling bearing fault feature extraction method based on LMD and envelope demodulation. Tian et al.^[2] combined the permutation entropy and the local mean decomposition method and applied to the fault diagnosis of bearing. Cheng et al.^[3] proposed a rolling bearing diagnosis method based on local mean decomposition and neural network to diagnose the fault vibration signal of rolling bearing. Yan et al.^[4] proposed a method of bearing composite fault diagnosis based on EEMD and Hilbert envelope analysis.

In recent years, the nonlinear analysis method based on the concept of entropy has been applied in the fields of medical, medicine biology, image processing and so on, such as approximate entropy, sample entropy, multi-scale entropy and fuzzy entropy. The permutation entropy algorithm is a new dynamic mutation detection algorithm, which can reflect the complexity and the irregular degree of one dimensional time series, and reflect the small change of time series data. Feng et al.^[5] combined the permutation entropy with the wavelet correlation filtering method and the hidden Markov model, and proposed the wavelet correlation permutation entropy, which is verified in the diagnosis of gear, bearing and other mechanical faults. Zheng et al.^[6] proposed a rolling bearing fault diagnosis method based on local feature scale decomposition and permutation entropy. Cheng et al.^[7] applying VPMCD to the fault diagnosis of rolling bearing based on the entropy method. Zhou et al.^[8] proposed fault diagnosis method based on permutation entropy and continuous hidden Markov model for gear fault diagnosis, which is based on permutation entropy and CHMM.

The actual signal is often mixed with a lot of noise, the noise signal is not only involved in LMD decomposition, resulting in the original fault information and noise is not easy to extract, but also makes the LMD decomposition level increase, which may lead to the algorithm does not converge, increase the boundary effect, even will cause the LMD to break down the actual physical meaning, thus affecting the accurate diagnosis of the fault^[9]. In this paper, combined the wavelet packet decomposition with LMD, the LMD algorithm is used to decompose the fault signal sub

bands of wavelet packet denoising to get some PF components. Then, the permutation entropy of the components is calculated. Finally, the fault diagnosis is carried out by the SVM which is suitable for small sample classification. The experimental results show that this method is an effective method for rolling bearing fault diagnosis.

2 Theoretical background

2.1 wavelet packet analysis

In practice, noise interference is inevitable in the experimental data. The traditional signal denoising methods, such as low pass filtering, Fourier transform are not suitable for non-stationary signal. Because those methods are equivalent to the signal through a low-pass or band-pass filter, filter out the useful part in the filter smoothing process^[10]. Wavelet analysis is a kind of time frequency analysis method to be widely used in recent years.

Wavelet packet analysis can be used to divide the frequency band of the signal, which has good local analysis ability. The suitable wavelet basis is selected to carry out the N layer wavelet packet decomposition on the signal. Then, quantify the threshold of the coefficients of each layer. Finally, the processed wavelet packet coefficients are reconstructed to obtain the signal after noise reduction.

The method of wavelet packet decomposition is in the following:

$$\begin{cases} d_{i,j,2m} = \sum_k h(k-2i)d_{k,j+1,m} \\ d_{i,j,2m+1} = \sum_k g(k-2i)d_{k,j+1,m} \end{cases} \quad (1)$$

Wavelet packet reconstruction is in the following

$$d_{i,j+1,m} = \sum_k h(i-2k)d_{k,j,2m} + \sum_k g(i-2k)d_{k,j,2m+1} \quad (2)$$

Where: $d_{i,j,m}$ is the wavelet packet coefficients, $h(k)$ and $g(k)$ are the expansion coefficients.

2.2 Local mean decomposition method

LMD is a new time frequency analysis method for adaptive non-stationary signals. The method separates the pure FM signal and the envelope signal from the original signal, and each envelope signal is multiplied with each frequency modulation signal to obtain a PF component. For any signal $x(t)$, it can be decomposed as follows.^[11]:

1) All local extreme points of the signal $x(t)$ are determined, and the local mean function $m_{11}(t)$ and envelope estimate function $a_{11}(t)$ are obtained by the operation and smooth processing:

$$\begin{cases} m_i = \frac{n_i + n_{i+1}}{2} \\ a_i = \frac{n_i - n_{i+1}}{2} \end{cases} \quad (3)$$

Where: n_i is the local extremum point, m_i is the Mean value of local extremum, a_i is the Envelope estimation.

2) Then the local mean function $m_{11}(t)$ is separated from the original signal $x(t)$ in the following

$$h_{11}(t) = x(t) - m_{11}(t) \quad (4)$$

The index in the formula indicates the sequence of the envelope signal and the iteration sequence.

3) Demodulation $h_{11}(t)$ to get the pure FM signal $s_{11}(t)$, and repeat the process until the local envelope function of the frequency modulation signal $s_{1n}(t)$ meet $a_{1(n+1)}(t) = 1$ as follows

$$s_{11}(t) = \frac{x(t) - m_{11}(t)}{a_{11}(t)} \quad (5)$$

4) The envelope signal of the PF component is obtained by multiplying all the local envelope functions in the iteration process as follows

$$a_1(t) = a_{11}(t) a_{12}(t) \cdots a_{1n}(t) = \prod_{q=1}^n a_{1q}(t) \quad (6)$$

5) The first PF component is obtained by multiplying the envelope signal with the pure FM signal as follows

$$PF_1 = a_1(t) \cdot s_{1n}(t) \quad (7)$$

6) After $PF_1(t)$ is separated from the original signal $x(t)$, the residual signal $u_1(t)$ is obtained. Repeating the above process k times, until $u_k(t)$ is a monotonic function in the following:

$$\begin{cases} u_1(t) = x(t) - PF_1(t) \\ u_2(t) = u_1(t) - PF_2(t) \\ \vdots \\ u_k(t) = u_{k-1}(t) - PF_k(t) \end{cases} \quad (8)$$

Then the original signal is decomposed into k components and a monotonic function is in the following

$$x(t) = \sum_{p=1}^k PF_p(t) + u_k(t) \quad (9)$$

The complete time-frequency distribution of the original signal can be obtained by combining the instantaneous amplitude and instantaneous frequency of all PF components.

2.3 Permutation entropy feature extraction

Permutation entropy (PE) is a new method to measure a random nature of time series and to detect an the abrupt change of complex dynamical systems. The algorithm is relatively sensitive to the change in signal data. The change of entropy can reflect and amplify the dynamic characteristics of the system, so it is widely used in various time sequence.

1) The discrete time series $X = \{x(i), i = 1, 2, \dots, n\}$ are obtained by the iteration of the system equations. According to the Takens theory the discrete time series is reconstructed X and the obtained the matrix is in the following^[12]:

$$\begin{bmatrix} x(1) & x(1 + \tau) & \cdots & x(1 + (m - 1)\tau) \\ x(2) & x(2 + \tau) & \cdots & x(2 + (m - 1)\tau) \\ \vdots & \vdots & \vdots & \vdots \\ x(j) & x(j + \tau) & \cdots & x(j + (m - 1)\tau) \\ \vdots & \vdots & \vdots & \vdots \\ x(K) & x(K + \tau) & \cdots & x(K + (m - 1)\tau) \end{bmatrix} \quad (10)$$

Where $j=1,2,\dots,K$; m is the embedding dimension (unit), τ is the delay time (unit), K is the number of reconstructed vectors, $K=n-(m-1)\tau$.

2) Each row in the reconstruction matrix is used as a component, then the component elements in ascending rearrange:

$$x(j + (q_1 - 1)\tau) \leq x(j + (q_2 - 1)\tau) \leq \cdots \leq x(j + (q_m - 1)\tau) \quad (11)$$

Where, q_1, q_2, \dots, q_m is the index for each element of the reconstruction component.

3) Then the arbitrary line component in ascending order rearrangement, a set of symbol sequence can be obtained in the following

$$S(j) = (q_{1j}, q_{2j}, \dots, q_{mj}) \quad (12)$$

4) Reconstruct vector to meet $K = m!$, m is the different symbol sequences of the reconstruction matrix. Calculating the emergence probability $P_1, P_2, \dots, P_j, \dots, P_K$ of $S(j)$.

5) According to Shannon's entropy theory, the permutation entropy $H_p(m)$ of time series X is defined as follows

$$H_p(m) = - \sum_{j=1}^K P_j \ln P_j \quad (13)$$

6) When $P_j = 1/m!$, the permutation entropy $H_p(m)$ get the maximum value. $\ln(m!)$ is usually used to

normalize $H_p(m)$, that is

$$0 \leq H_p = H_p(m) / \ln(m!) \leq 1 \quad (14)$$

Where, H_p is the normalized permutation entropy, which reflects the degree of the time series X .

The greater H_p , the more random the time series is; the smaller H_p , the more regular the sequence, and can also reflect and magnify the minute changes in the time series.

3 Fault diagnosis based on MPD and LMD permutation entropy

In the early stage of bearing failure, the frequency component of the bearing damage position is characterized by weak energy, which is usually masked by noise interference. In order to eliminate the influence of random noise and local strong interference on the feature extraction, and extract the effective fault index from the vibration signal of the bearing. The rolling bearing fault diagnosis process based on wavelet packet decomposition and LMD sample entropy is shown in Figure 1.

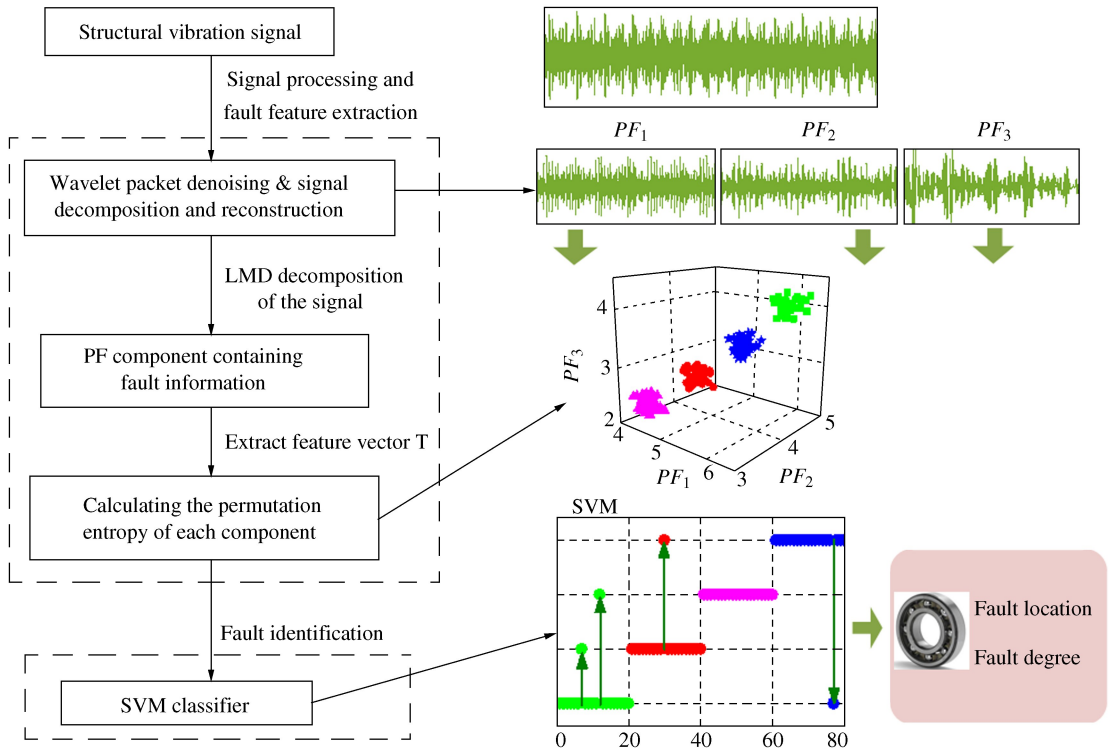


Figure 1 Flow chart of fault diagnosis

The specific steps are as follows:

1) Wavelet threshold method is used to eliminate noise of the original vibration signal, and the spectrum of vibration signal after de-noising is analyzed.

- 2) According to the fault characteristic frequency of bearing and the signal energy concentration section, determine the wavelet base and the decomposition level, and the N layer wavelet packet decomposition is performed on the denoising signal. Wavelet packet reconstruction of the characteristic frequency band signal.
- 3) The local mean decomposition of the reconstructed signal is carried out, and a number of PF components is obtained.
- 4) Choose the previous five PF components as the research object, and calculate the permutation entropy, which will be used as the feature vector $T = [PE_1, PE_2, PE_3, PE_4, PE_5]$.
- 5) The extracted permutation entropy feature vector is input to multiple fault classifier to train the support vector machine. Then, the test samples are input into the trained SVM, and the fault type and fault degree of the bearing are determined by the output of the classifier.

4 Numerical example

In order to verify the effectiveness of the proposed method, the vibration data of rolling bearing of Case Western Reserve University in the United States is measured. The test bearing is installed on the motor drive end, which is the SKF 6205-2RS deep groove ball bearing. The damage condition of the bearing is a single local defect which is formed by the electric spark machining technology. In this paper, the motor speed is 1 797 r/min, fault diameter is 0.177 8 mm, and sampling frequency is 12 kHz. The vibration signals of four kinds of bearings are collected respectively: the normal state, the inner ring wear, the outer ring wear and the rolling body wear. Each state takes 40 sets of data samples, each sample length is 2 048.

4.1 Fault signal feature extraction

The local mean decomposition method is sensitive to noise. In order to eliminate the influence of noise on the diagnosis results, wavelet packet threshold denoising is performed on the original signal, and obtain the time domain waveform is the vibration acceleration signal, as shown in Figure 2. Time domain waveform has obvious impact, but the waveform is more complex, which is not easy to distinguish the working status of the bearing only through the time domain waveform. Figure 3 shows the spectrum of the vibration signal when the inner and outer ring of the bearing is in fault. It can be seen that the energy of internal and external fault vibration signals are mainly concentrated in 0~1.5 kHz and 2.5~3.5 kHz, which indicates that the frequency characteristic of the

fault signal is mainly located in the frequency band. The 3 layer wavelet packet decomposition of the bearing vibration signal is carried out to obtain the reconstructed signal of the characteristic frequency band, which improves the accuracy of the local mean decomposition.

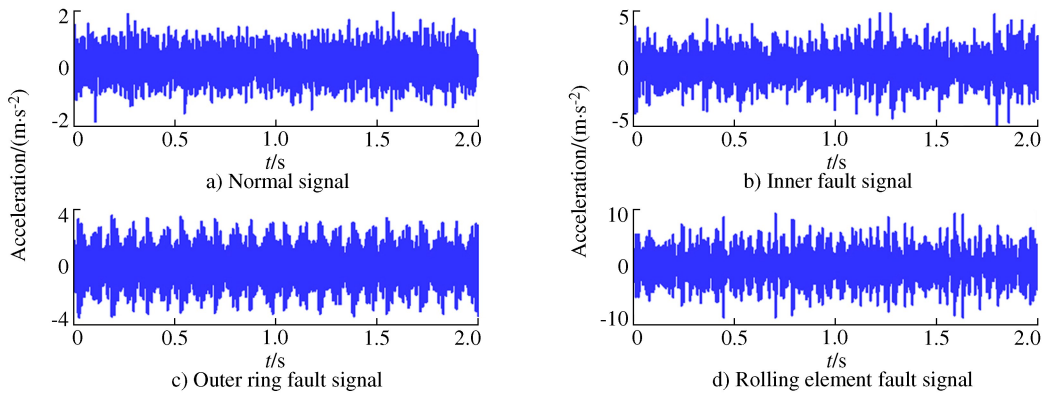


Figure 2 Time domain waveform of the fault vibration signal after de-noising

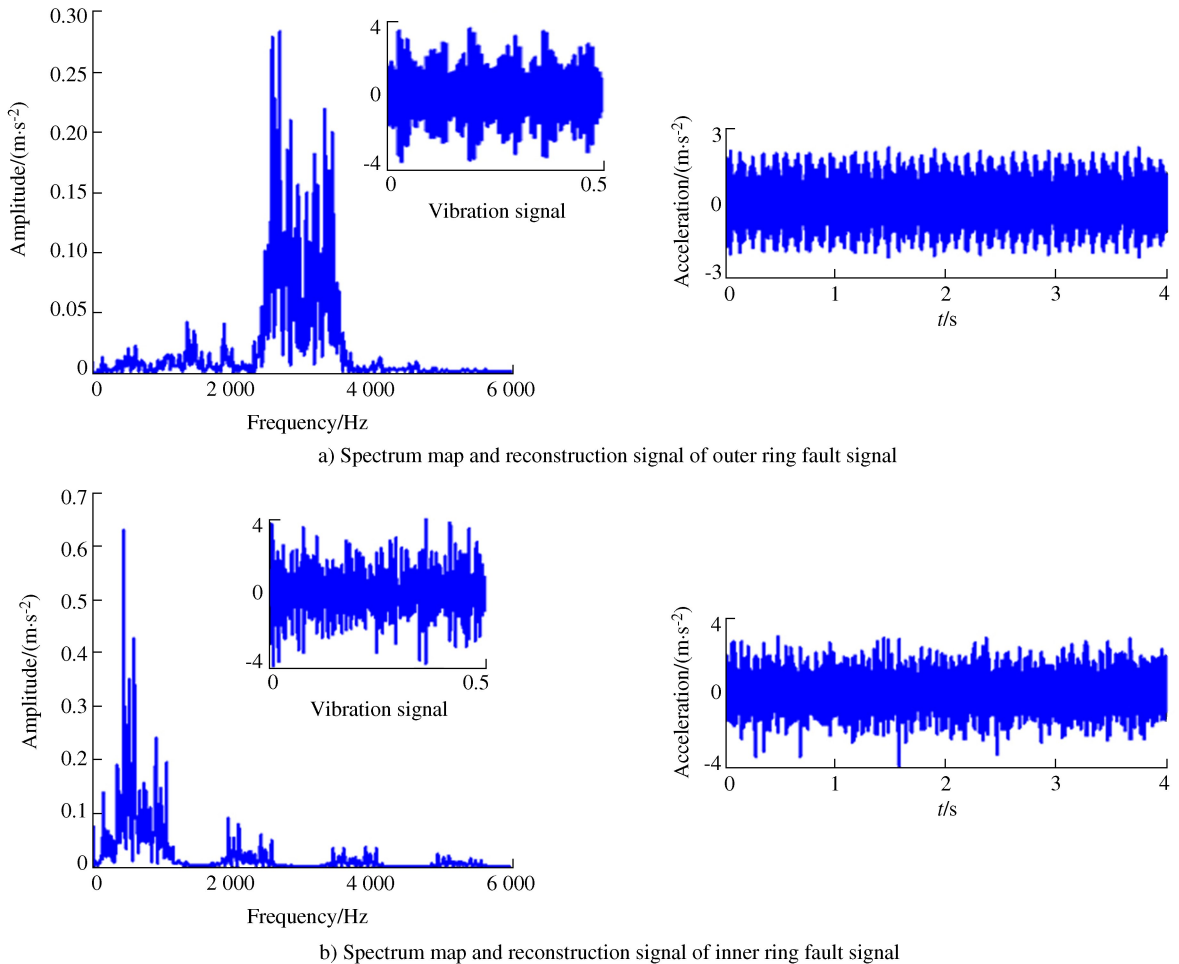


Figure 3 The spectrum and the reconstructed signal of the fault signal of the inner and outer ring of the bearing

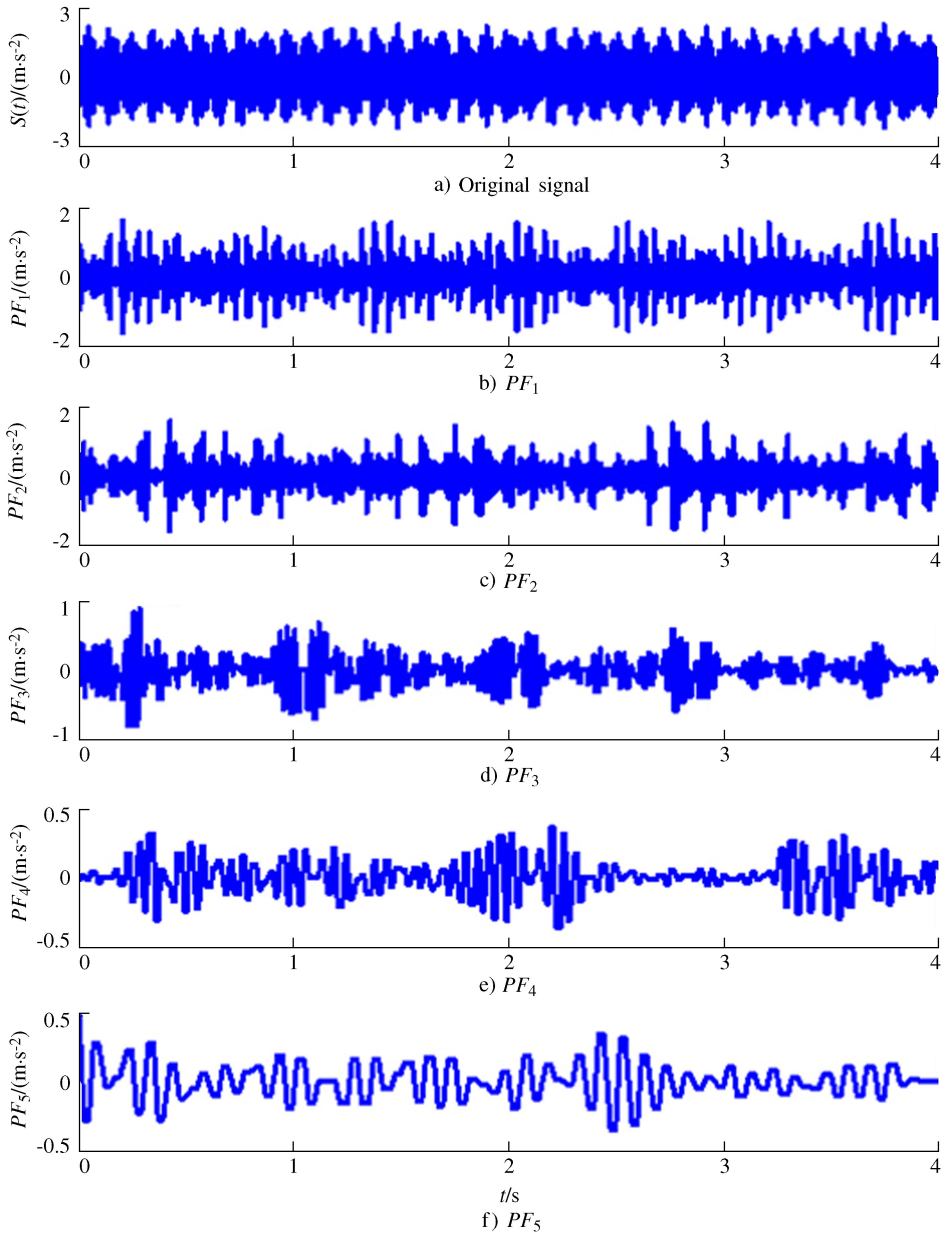


Figure 4 LMD decomposition results of the outer ring fault signal

The local mean decomposition of the reconstructed fault signal is carried out, and obtain some PF components with certain eigenvalues. As shown in Figure 4, the decomposition of the PF component highlights the local characteristics of the data. In order to avoid the long sample data of the feature vector, and simplify the classifier structure, we select the previous five PF components which contain the main fault information, and calculate their permutation entropy.

There are two main parameters in the computation of permutation entropy: embedding dimension m and delay time τ , and the embedding dimension m is usually taken $3 \sim 7$. If m is too small, the algorithm loses its

effectiveness, and cannot detect the sequence of dynamic mutation; if the m is too large, not only the calculation is time-consuming, and cannot reflect the subtle changes in the sequence^[6, 13]. The mutual information method and the pseudo nearest neighbor method are given for determining m and τ in the literature^[14], and the influence of time delay on time series calculation is small.

In order to verify the validity of the method, the bearing fault signal is directly decomposed by LMD and EMD, respectively. The permutation entropy of the decomposition result is used as the training sample set for the feature vector, and the fault recognition results of the 3 methods are compared. The typical feature vectors of each fault are shown in Table 1.

Table 1 Typical feature vectors of different feature extraction methods

Bearing type	WPD & LMD			LMD			EMD		
	PF_1	PF_2	PF_3	PF_1	PF_2	PF_3	PF_1	PF_2	PF_3
Normal state	6.156	4.987	3.864	6.428	5.735	4.498	6.256	4.856	3.549
Inner ring wear	4.735	3.518	2.427	4.719	3.571	2.406	4.742	3.412	2.215
Outer ring wear	4.106	3.193	2.081	3.934	3.324	2.158	4.279	2.256	1.598
Rolling body wear	5.431	4.184	3.162	5.648	4.248	3.176	5.781	4.602	3.365

As seen from the Table 1, the statistical characteristics of each component of the 4 conditions are obviously different. Due to the randomness and uncertainty of the signal when the bearing is in the normal state, the permutation entropy of the signal component is the largest. When the bearing failure occurs, the random and dynamic behavior of the signal will be changed. The outer ring of the bearing is relatively fixed, so the entropy is smaller; the rolling motion is irregular, the entropy is slightly larger than the inner and outer rings, while can not directly judge of bearing fault location through the entropy. Therefore, in order to realize the fault diagnosis, a multi fault classifier based on support vector machine is established.

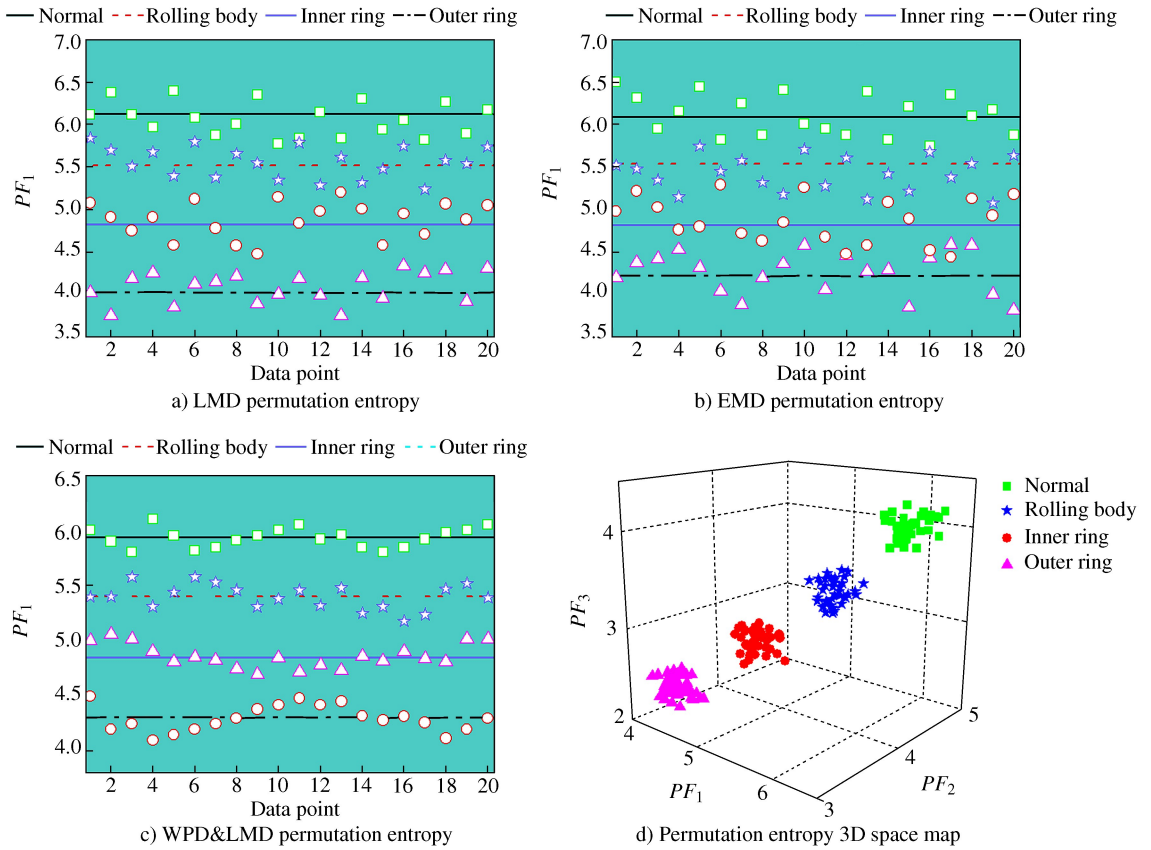


Figure 5 Permutation entropy distribution of 3 kinds of methods

Figure 5a)~5c) indicates that the permutation entropy envelope distribution of the first component (that is IMF_1 or PF_1), respectively, after the fault signal is decomposed based on the LMD, EMD and WPD_LMD methods. It can be seen that the end effect of EMD decomposition is very obvious, and there is a serious oscillation of the envelope in the vicinity of the end. Figure 5d) is the permutation entropy of the previous three PF components (that is, PE_1 , PE_2 , PE_3) based on the wavelet packet and the LMD decomposition method, formed a three-dimensional feature vector set. From the spatial distribution of entropy can be seen that the extraction of WPD_LMD entropy does not exist aliasing phenomenon in the three-dimensional space, showing a good intra class aggregation and inter class separation. Therefore, it is proved that the method of fault feature extraction based on wavelet packet and LMD decomposition is superior and effective.

4.2 Bearing fault diagnosis based on SVM

The generalization ability of SVM is strong, and it can solve the problem of high dimension, small sample size and nonlinear pattern classification. A support vector machine with "one to one" multi-classification algorithm is used to establish the model of bearing fault diagnosis. The kernel function is radial basis function, kernel

parameter $\sigma=3.4$, penalty parameter $C=2.25$.

40 groups of the four kinds of working conditions are selected to form the sample set, and randomly selected each state of the 20 sets of feature vectors for SVM training, to achieve automatic classification of bearing working state. The remaining 20 sets of data are used as test samples to test the validity of the model, as shown in Table 2.

Table 2 Bearing data classification

Bearing states	Training sample	Testing sample	Category label
Normal state	20	20	1
Inner ring wear	20	20	2
Outer ring wear	20	20	3
Rolling body wear	20	20	4

Using SVM to classify the bearing different working states, the test results of the 3 methods are shown in Table 3. For the same finite number of samples, the recognition rate of bearing fault based on wavelet packet decomposition and LMD permutation entropy is significantly higher than the feature vector of the fault signal based on LMD or EMD. The recognition rate of the bearing states is normal 90%, the inner fault 95%, the outer ring fault 100%, the rolling element fault 90%, the average recognition rate is 95%, respectively, and the classification results are shown in Figure 6. Therefore, the superiority of the proposed method is verified in terms of fault feature extraction and fault identification.

Table 3 Results of fault location identification by different methods of extraction

Bearing state	WPD-LMD		LMD		EMD	
	Accuracy	Average	Accuracy	Average	Accuracy	Average
Normal state	90%		85%		85%	
Inner ring wear	95%		90%		80%	
		95%		85%		83.75%
Outer ring wear	100%		85%		90%	
Rolling body wear	95%		80%		80%	

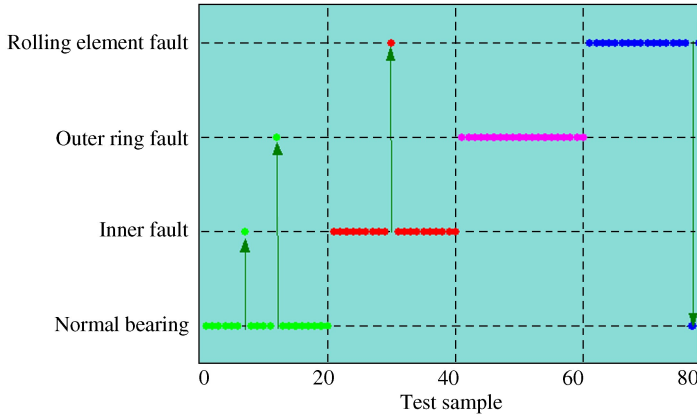


Figure 6 Classification results of the bearing different working states

In order to further verify the effectiveness of the proposed method in this paper, the different faults of the bearing inner ring are classified. The fault diameter is 0.177 8 mm, 0.355 6 mm, 0.533 4 mm, 0.711 2 mm, respectively. According to the same steps, obtaining the entropy feature vector, building a support vector machine classification, and the classification results as shown in Figure 7. It can be seen that the total recognition rate is 93.75%, which shows that the method can effectively identify the fault of the rolling bearing in different locations, different degree of damage, and improve the diagnostic accuracy and recognition ability.

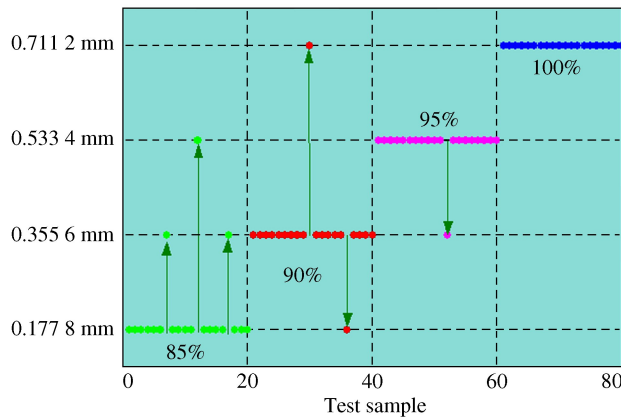


Figure 7 Classification results of different fault degree of bearing outer ring

5 Conclusions

The bearing fault information is easy to be submerged in the background noise and interference, so that the signal feature extraction becomes very difficult. In this paper, wavelet packet decomposition is combined with LMD permutation entropy, and SVM is used as classifier to identify fault types. The following conclusions can be drawn

from the experimental results:

- 1) The wavelet packet analysis method not only can eliminate the noise effectively, but also can extract the fault information sub band, which can improve the accuracy of the LMD decomposition, so that the fault feature extraction is more adequate and accurate. The combination of wavelet packet decomposition and LMD can effectively extract fault features and overcome the limitations of traditional methods.
- 2) The permutation entropy can be used to detect the random and dynamic behavior of vibration signals. The LMD permutation entropy of vibration signal in the same fault condition is presented in $PF_1 > PF_2 > PF_3 > PF_4 > PF_5$, which is consistent with the waveform change after LMD decomposition. The permutation entropy can be better matched with the mechanical fault signals, which indicates the characteristics of mechanical failure.
- 3) The feature extraction method based on wavelet packet decomposition and LMD permutation entropy is better than the directly LMD or EMD decomposition. Experimental results show that the proposed method is feasible and can effectively classify the bearing condition and fault degree, and can be used as an effective method for fault identification.

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