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Interesting Beat Phenomenon for Airfoil

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Abstract: The airfoil with two degrees is simulated to get the beat phenomenon. The results indicate that the occurrence of beat phenomenon is very sensitive to the equivalent frequency and damping, contributed by the structural and aerodynamic ones. Only the equivalent damping approaches to zero and the equivalent frequency is very close to the gust frequency which the beat phenomenon occurs.

Keywords: beat phenomenon; aeroelasticity; gust; flutter; airfoil

1 Introduction

Aeroelasticity focuses on the study of the stability and response of elastic structures undergoing aerodynamic forces. Typical aeroelasticity involves the coupling of elasticity, inertial force, and aerodynamics. In the aeroelastic response problem, the excitation comes usually from turbulence or gust, missile or cargo release, etc.

The classical beat phenomenon can appear in many engineering vibrating system[1-3]. Questions come out that which parameters cause the beat phenomenon in the aeroelastic systems and how to estimate the gust alleviation effect when the beat phenomenon exists. Therefore, the mechanism of

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beat phenomenon is investigated to explain the specific aeroelastic phenomenon. Firstly, the analytic solution of beat phenomenon for a single degree of freedom system is obtained. Secondly, the beat phenomenon of a wing section due to gust is simulated with a linear quasi-steady aerodynamic model, to reveal the importance of "equivalent damping" on the beat phenomenon.

2 Beat phenomenon in one-dimensional forced vibrating system

A vibrating system of a single degree-of-freedom is represented by using mass-spring with a damper, shown in Figure 1. The system is drived by an external force $F(t)$.

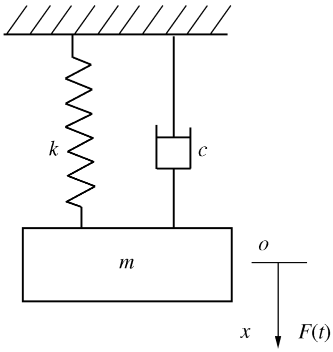


Figure 1 Schematic diagram of single degree-of-freedom vibrating system

Its dynamic motion equation is written as follows:

$$m\ddot{x} + c\dot{x} + kx = F(t) \tag{1}$$

Where $F(t) = B\sin(\omega t)$.

With an initial displacement x_0, the dynamic displacement of this vibrating system due to external sinusoidal force is written as follows:

$$x(t) = e^{-\xi\omega_0 t}\left(x_0\cos(\omega_d t) + \frac{\dot{x}_0 + \xi\omega_0 x_0}{\omega_d}\sin(\omega_d t)\right) + B\beta e^{-\xi\omega_0 t}\left(\sin\theta\cos(\omega_d t) + \frac{\xi\omega_0\sin\theta - \omega\cos\theta}{\omega_d}\sin(\omega_d t)\right) + B\beta\sin(\omega t - \theta) \tag{2}$$

$$
\begin{cases}
\beta(\omega) = \dfrac{1}{\sqrt{(1-\bar{\omega}^2)^2 + (2\xi\bar{\omega})^2}} \\
\theta(\omega) = \arctan \dfrac{2\xi\bar{\omega}}{1-\bar{\omega}^2} \\
\bar{\omega} = \dfrac{\omega}{\omega_0} \\
\omega_0 = \sqrt{\dfrac{k}{m}}
\end{cases}
\tag{3}
$$

The displacement solution in Equation (2) can be divided into three terms. The first term of the solution is the transient response caused by the initial condition, which is the same as the solution of a free-vibration system. The second term is also a transient solution, while it is caused by the forced excitation, not the initial displacement. Usually, the responses of these two terms vanish with time, due to the positive damping ration of ξ in real engineering structures. Hence, only the third term of the solution retains with time, it vibrates along with the forced input.

The beat phenomenon comes from the second term. The condition for beat phenomenon is that the natural frequency approaches to the excitation frequency[4]. Let $\bar{\omega}=1+2\varepsilon$, and ε is a very small value. Without considering the damping ratio and the initial conditions, then dynamic displacement in Equation (2) can be simplified to the following:

$$
x(t) \approx -\frac{B}{2\varepsilon}\sin(\varepsilon\omega_0 t)\cos(\omega_0 t) \tag{4}
$$

It is called the "beat phenomenon" with the periodic amplitude of $\frac{B}{2\varepsilon}\sin\varepsilon\omega_0 t$ and frequency of ω_0, shown in Figure 2. Only in this case, the beat phenomenon can exist permanently, not vanish with time.

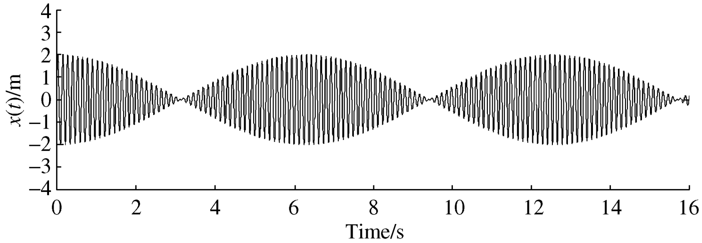


Figure 2 Beat phenomenon of a SDOF vibrating system

Actually, the beat phenomenon ceases after several transient time periods, since the engineering structures have positive damping. From previous research, the conditions for beat phenomenon in dynamic forced systems are those: firstly, the excitation frequency is close to the natural frequency of the vibrating system; secondly, the damping of the vibrating system is as small as possible. However, zero damping is an ideal case, in this situation the beat phenomenon will not vanish with time.

3 Beat phenomenon in linear aeroelastic systems due to gust

3.1 Aeroelastic equation for wing section with discrete gust excitation

When a wing section with plunge and pitch degrees of freedom is undergoing a discrete gust, the effective angle of attack can be written as

$$\alpha_V = \alpha + \frac{1}{V}\dot{h} + \left(\frac{1}{2} - a\right)\frac{b}{V}\dot{\alpha} + \frac{1}{V}w_g \tag{5}$$

Whese w_g is the discrete gust velocity at the airfoil location. In the wind tunnel test, it is usually generated as a sinusoidal formation, that is $w_g = A\sin(\omega t)$.

The schematic diagram of the wing section undergoing sinusoidal gust is shown in Figure 3.

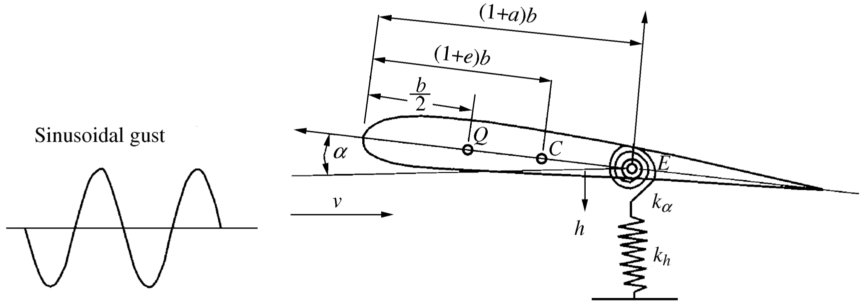


Figure 3 Schematic diagram of the wing section undergoing gust

With the quasi-steady aerodynamic model, the aeroelastic motion equation of the wing section is written as follows:

$$\rho V^2 b\begin{bmatrix} -c_{L\alpha} \\ -c_{m\alpha} b \end{bmatrix}\alpha_V = \begin{bmatrix} m & m x_\alpha b \\ m x_\alpha b & I_\alpha \end{bmatrix}\begin{bmatrix} \ddot{h} \\ \ddot{\alpha} \end{bmatrix} + \begin{bmatrix} c_h & 0 \\ 0 & c_\alpha \end{bmatrix}\begin{bmatrix} \dot{h} \\ \dot{\alpha} \end{bmatrix} + \begin{bmatrix} k_h & 0 \\ 0 & k_\alpha \end{bmatrix}\begin{bmatrix} h \\ \alpha \end{bmatrix} \tag{6}$$

The equations describe the complete aeroelastic system with gust excitation. The degrees of freedom of the wing section are described by the plunge, h, and the pitch, α. The left side of Equation (6) is the aerodynamic force and moment due to the elastic motion and gust excitation. The physical parameters of this wing section are shown in Table 1.

Table 1 Parameters of the airfoil

Parameter	Value	Parameter	Value
b/m	0.135	ρ/(kg · m^{-3})	1.225
m/kg	12.387	x_α	0.246 6
I_α/(m^2 · kg)	0.065	k_h/(N · m^{-1})	2 844.4
c_α/(m^2 · kg · s^{-1})	0.180	c_h/(kg · s^{-1})	27.43
$c_{L\alpha}$	6.28	$c_{m\alpha}$	−0.628
k_α/(Nm · rad^{-1})	2.82	a	−0.6

The natural frequencies of the pitch mode and the plunge mode are 6.6 rad/s and 15.2 rad/s. By introducing the state space variables, Equation (6) can be arranged in a more compact form.

$$\boldsymbol{A}\boldsymbol{x} = \boldsymbol{B}\boldsymbol{x} + \boldsymbol{f}(u) \tag{7}$$

$$\boldsymbol{x} = \begin{bmatrix} \dot{h} \\ \dot{\alpha} \\ h \\ \alpha \end{bmatrix}, \quad \boldsymbol{A} = \begin{bmatrix} m & mx_\alpha b & 0 & 0 \\ mx_\alpha b & I_\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \boldsymbol{f}(u) = \begin{bmatrix} -\rho V^2 bc_{L\alpha}\dfrac{1}{V} \\ \rho V^2 b^2 c_{L\alpha}\dfrac{1}{V} \\ 0 \\ 0 \end{bmatrix} w_g$$

$$\boldsymbol{B} = \begin{bmatrix} -\rho V^2 bc_{L\alpha}\dfrac{1}{V} - c_h & -\rho V^2 bc_{L\alpha}\left(\dfrac{1}{2} - a\right)\dfrac{b}{V} & -k_h & -\rho V^2 bc_{L\alpha} \\ \rho V^2 b^2 c_{m\alpha}\dfrac{1}{V} & \rho V^2 b^2 c_{m\alpha}\left(\dfrac{1}{2} - a\right)\dfrac{b}{V} - c_\alpha & 0 & -k_\alpha + \rho V^2 b^2 c_{m\alpha} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{8}$$

The characteristic matrix of Equation (7) is $\boldsymbol{A}^{-1}\boldsymbol{B}$. The eigenvalue of $\boldsymbol{A}^{-1}\boldsymbol{B}$ reveals the equivalent modal frequency and modal damping ratio of this linear aeroelastic system, composed by the structural ones and aerodynamic ones. The modal frequency and damping ratio can be calculated by the characteristic matrix.

The eigenvalues would change with the flow velocity, indicated in matrix $\boldsymbol{B}$. From the following section, it will be seen that the matrix $\boldsymbol{B}$ is important to the occurrence of beat phenomenon.

3.2 Simulation results and discussion

In the current study, a fixed step 4rd-order Runge-Kuta algorithm is employed to solve Equation (7) in order to calculate the gust response. The initial condition is set to be zero. The time step is $(\Delta t) = 0.001$. The gust velocity is set as a sinusoidal function, which is expressed as follows:

$$w_g = 0.2V\sin(\omega t) \tag{9}$$

In simulation, $\omega = 13$ rad/s. It is chosen to be close to the equivalent frequency of the pitch mode. The gust amplitude varies with flow velocity.

At different velocities, the pitch angle in the time domain is simulated, indicated in Figure 4. From Figure 4, at the velocity of 12.0 m/s, the pitch angle for a long time is nearly sinusoidal, similar as the gust profile. With the increasing of flow velocity, the beat phenomenon occurs at a velocity of 12. 11 m/s. When the flow velocity increases slightly, the pitch angle diverges. More investigations infer that "flutter" in aeroelasticity occurs at this velocity.

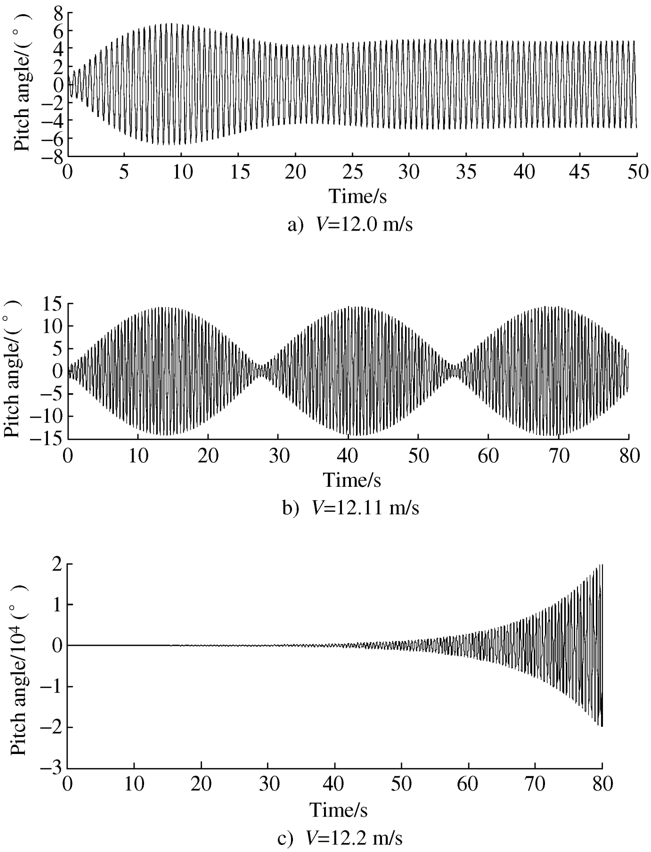
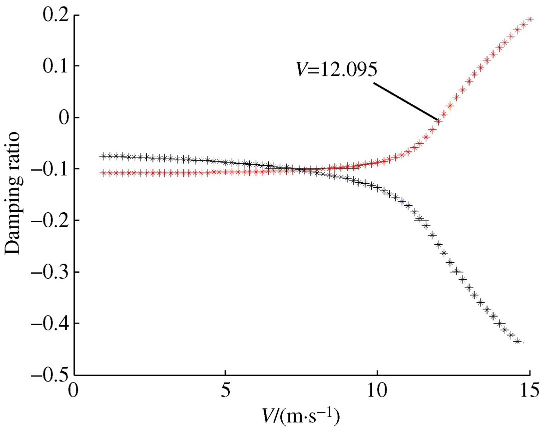
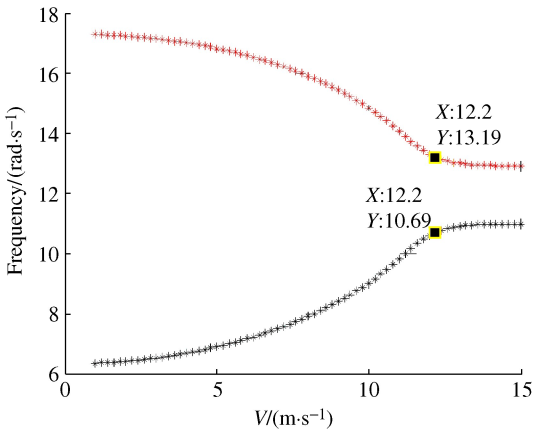


Figure 4 Pitch angles in the time domain at different flow velocities

In order to investigate the beat phenomenon in detail, the eigenvalue is calculated at different velocities. The real part of the eigenvalue is the equivalent damping ratio, while the imagination part of the eigenvalue is the radius frequency of one specific mode. From the expression of state-space matrix $\boldsymbol{A}^{-1}\boldsymbol{B}$, the aerodynamic force would significantly affect the damping ratio. It makes the damping ratio of the pitch mode approaching to zero damping at the flutter speed, shown in Figure 5. Together with the above conclusion, the beat phenomenon can only occurs at the critical flutter speed. In this situation, the equivalent damping ratio of the pitch mode, which is composed by the positive structural damping ratio and the negative aerodynamic damping ratio, is nearly zero at the speed.



a) V-G diagram



b) V-F diagram

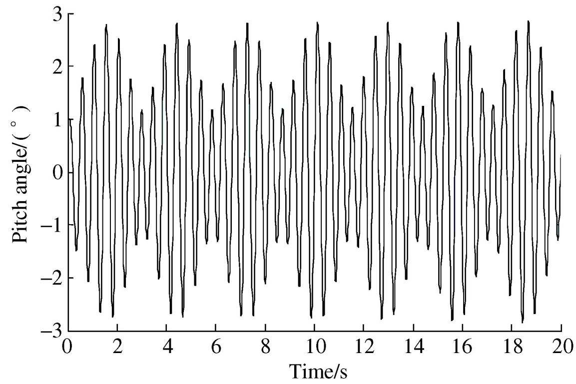
Figure 5 The damping ratio and modal frequency versus the flow velocity

From the above analysis, the condition for beat phenomenon of a linear aeroelastic system is also the flutter condition. That is: the mass distribution, the stiffness of the normal modes should satisfy the requirements for typical flutter. More simulations are conducted under different coupling mass, structural damping ratio, natural frequencies. Results indicate that the velocity that beat phenomenon occurrence would change and the beat frequency also varies. However, the velocity of beat phenomenon is still the same as the flutter velocity. It means for a linear aeroelastic system, beat phenomenon is seldom observed in the wind tunnel test or flight test[2], since the dangerous flutter would occur at that velocity.

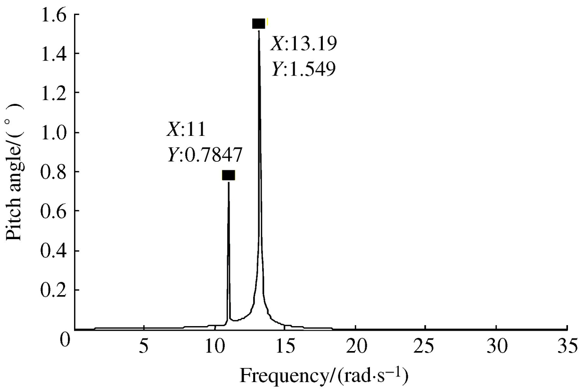
Note that the beat phenomenon due to gust excitation is not same as the phenomenon due to initial conditions in the section 1. In the latter condition, the beat phenomenon is related with the modal frequencies of the two normal modes and with the coupling mass. While in the former situation, beat phenomenon happens certainly at the flutter velocity when the gust frequency is close to the frequency of the flutter mode, no matter the gap of the modal frequencies is large or the gust amplitude is small.

From the *V-F* diagram in Figure 5 and the response in Figure 4, beat phenomenon is obvious when the gust frequency is close to the frequency of the pitch mode. In Figure 5, there is also a plunge mode with a frequency of 10. 9 Hz at a flutter velocity of 12. 1 m/s. In order to check whether the plunge mode contributes to the beat phenomenon, the gust frequency is set as 11 rad/s, which is close to the frequency of the plunge mode. The pitch angle in the time domain and in the frequency domain is shown in Figure 6. The peak frequencies are 11 rad/s and 13. 19 rad/s. Those are the gust frequency and the pitch frequency. It infers that though the gust frequency is close to the frequency of the plunge mode, the beat phenomenon is not caused by the plunge mode. The reason can be found back in Figure 5. At the flutter velocity, only the damping of the pitch mode is zero. The damping of the plunge mode is still a positive value. Hence, the transient response of the plunge mode may vanish. However, the transient response of the pitch mode still exists.

Therefore, the conclusion for linear aeroelastic gust response can be strengthened as: only the gust frequency is close to the frequency of the flutter mode, beat phenomenon can exist at the critical flutter speed.



a)



b)

Figure 6 Gust response at the velocity of 12. 1 m/s with the gust frequency of 11 rad/s

4 Conclusions

From beat phenomenon in engineering vibration problems, we simplify it to a linear aeroelastic system with two DOFs due to discrete gust. From the simulation results, it is concluded that:

1) The conditions for beat phenomenon in a linear aeroelastic system under a sinusoidal excitation include two aspects. First, the equivalent damping ratio, composed of the structural and the aerodynamic damping, is zero. Second, the excitation frequency is close to the equivalent modal frequencies.

2) The beat phenomenon can only occur at the critical flutter velocity under an excitation with

a flutter "cross-mode" frequency.

3) The above two strict conditions infer that beat phenomenon is seldom observed in a linear aeroelastic system.

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Brief Biography

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