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Taper Analysis of Work-piece with Lathe Spindle Angular Swinging

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Abstract: This thesis focuses on a general lathe spindle, quantitatively analyzes the machining precision with spindle swinging in geometry way and derives a mathematical expression of machining errors at different swing frequencies. By applying MATLAB to simulate the work pieces´ cross-section at different swing frequencies, taper is analyzed and variation law is summarized.

Key words: spindle; angular swing; swing frequency; MATLAB; taper

1 Introduction

In the machine tool design and precision machining, spindle system in machine is the key to guarantee the machining accuracy. As the dominant vibration system among the multiple degree of freedom vibration systems, the error motion of a machine tool spindle includes the radial run-out, axial shifting and angular swinging. In the most of available literatures, they analyze the spindle angle and machining error with the assumption that the spindle swings and rotates at a same frequency. However, in the actual production, the swinging frequency of the main spindle is generally inconsistent with the rotation frequency, and few of paper has ever discussed it and been published. Therefore, this article focuses on discussing the processing error of mathematics and geometry in the main shaft plane with the swinging angles, then calculates and analyzes the work pieces´s taper and its influencing factors at a series of swing frequencies.

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2 Mathematical analysis and simulation at fixed swing frequencies

2.1 Mathematical expressions of spindle angular swinging

Establishing the static coordinate system $XYZO$ of the main spindle at first, and OZ is the ideal axis of the spindle. Then, establishing the dynamic coordinate system $X'Y'Z'O$, and shaft OZ' is the spindle instantaneous axis. The built spindle static/dynamic coordinate system is shown in Figure 1.

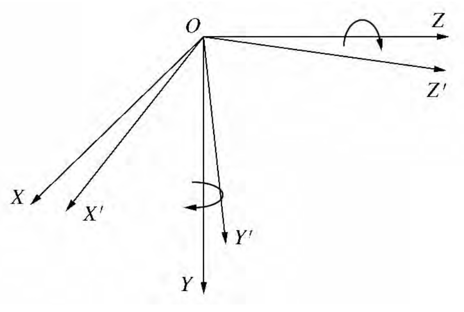

Figure 1 The spindle static/dynamic coordinate systems

The spindle rotates with the angular velocity ω, meanwhile, its axis line angularly swings at the same frequency in XOZ plane. The rule of its swinging is:

$$\theta = \theta_0 \cos\alpha \tag{1}$$

θ_0 is the swinging amplitude of the spindle axis; α is the spindle's rotation angle, $\alpha = \omega t$.

2.2 The parameter equation of the workpiece cross-section

Due to the spindle's rotation and angle swinging existing simultaneously, so we can obtain the relationship between the moving coordinate system and the absolute coordinate system by matrix transformation. Regard the spindle's rotation as the coordinate system $XYZO'$s rotation which around the OZ axis. Regard spindle' swing in XOZ plane as spindle's rotation around OY axis. After rotating twice, we can get

$$\begin{bmatrix} Z \\ X \\ Y \end{bmatrix} = \begin{bmatrix} 1 & -\theta\cos\alpha & \theta\sin\alpha \\ \theta & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} Z' \\ X' \\ Y' \end{bmatrix}$$

The shape of the lathed workpiece is decided by the relative trajectory of the tools in the dynamic coordinate, and the expression is as followed

$$\begin{bmatrix} Z' \\ X' \\ Y' \end{bmatrix} = \begin{bmatrix} 1 & \theta & 0 \\ -\theta\cos\alpha & \cos\alpha & \sin\alpha \\ \theta\sin\alpha & -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} Z \\ X \\ Y \end{bmatrix} \tag{2}$$

When turning, the tool's coordinate position in the absolute coordinate system is $Z = L$, $X = R$, $Y = 0$. Among them, L is the position of the cutting tool on the rail direction, and changes with the move tool; R is the manufacturing radius of the workpiece.

Plug in the tool position parameter and the formula (1) into the formula(2), then we can get the cutting tool's trajectory parameter equation in the moving coordinate system.

$$\begin{cases} Z' = L + R\theta_0\cos\alpha \\ X' = (R - L\theta_0\cos\alpha)\cos\alpha \\ Y' = (L\theta_0\cos\alpha - R)\sin\alpha \end{cases} \tag{3}$$

By formula (3) we obtain the instantaneous radius of the workpiece cross section, which is

$$r = \sqrt{X'^2 + Y'^2} = R - L\theta_0\cos\alpha \tag{4}$$

2.3 Trajectory analysis at fixed swing frequencies

Draw the above equation in polar coordinates with MATLAB. The spindle rotates one circle, and the cross section contour diagram is shown in Figure 2. Analysis shows that if the swinging frequency is equal to the rotational frequency, the profile of the workpiece is approximately a circle, although its center deviates from the ideal center after the rotation of the workpiece. Its roundness error is very small and negligible.

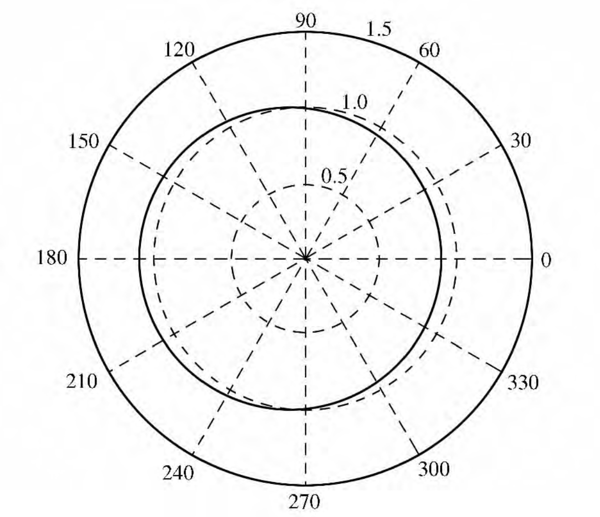

Figure 2 The trajectory of tool rotation when the swinging frequency = rotational frequency

3 Mathematical analysis and simulation at different swing frequencies

3.1 Parameter equation of the track at different swing frequencies

When the spindle swinging frequency is equal to the spindle rotation frequency, $r = R - L\theta_0\cos\alpha$ is given by the formula(4).

Then calculate:

When the spindle swinging frequency is 2 times of the spindle rotation frequency, $r=R-L\theta_0\cos(2\alpha)$;

When the spindle swinging frequency is 3 times of the spindle rotation frequency, $r=R-L\theta_0\cos(3\alpha)$;

Similarly, when spindle swing frequency is n times of the spindle rotation frequency, $r=R-L\theta_0\cos(n\alpha)$.

3.2 Trajectory and variation law at different swinging frequencies

3.2.1 Trajectory and regularity analysis at low swing frequencies

With the spindle swinging frequency going low, it becomes smaller than the spindle rotation frequency, which means the value of n is less than 1. In order to visually analyze cross-sectional profile of the workpiece, assume that $R=1$, $L=1$, $=0.1$, $n=0.6$. After the spindle rotating one circle, the cross section contour diagram is shown in Figure 3.

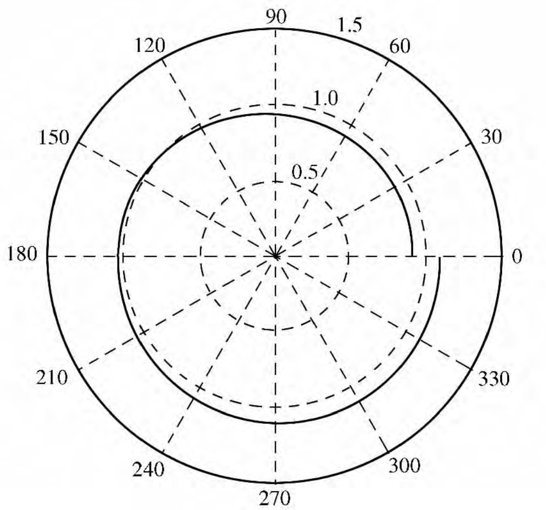

Figure 3 The trajectory of workpiece rotation when the swinging frequency = 0.6 times rotational frequency

From the figure above, we can see that the cross section contour is a non-closed curve but the radius is regularly changing.

Keep the workpiece rotating m circles (m is an integer) until its contour curve closed. Then, the inner envelope is the outer contour of the workpiece as shown in Figure 4.

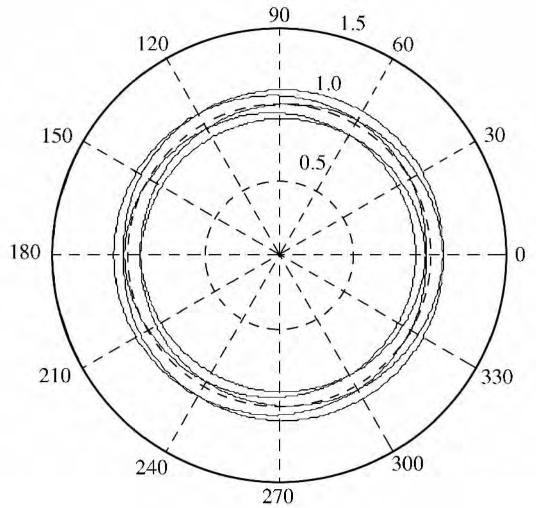

Figure 4 Curve closed after 5 turns

Extract the inner envelope, which is the actual outer contour of the workpiece during processing, as shown in Figure 5.

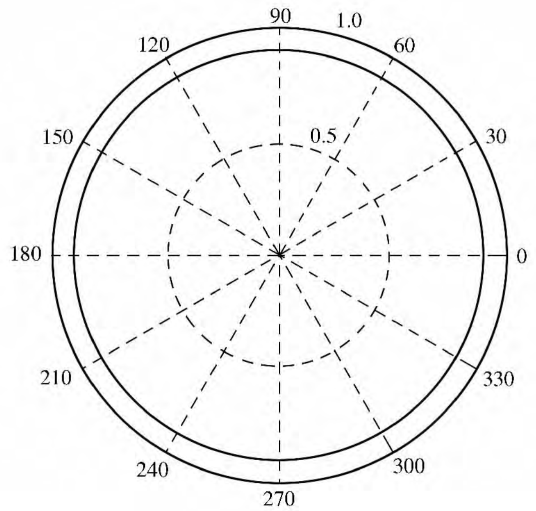

Figure 5 The inner envelope line

By observing, the shape of the inner envelope seems like a circle. But in fact the contour surface is corrugated as shown in Figure 6. In this case, the diameter of the outer contour is approximate to the diameter of the least-square circle.

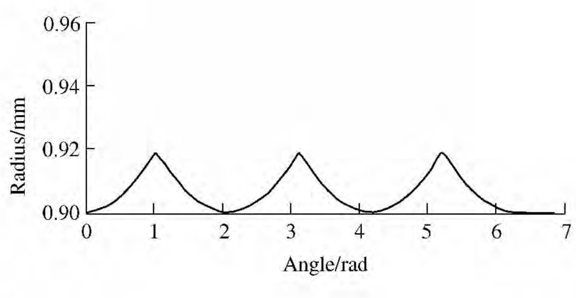

Figure 6 The outer contour radius variation

3.2.2 Trajectory and regularity analysis at high swing frequencies

When the spindle swinging frequency is high and greater than the spindle rotation frequency, it makes the value of n greater than 1. Assuming $n=3$ and the spindle one circle rotated, the cross section contour diagram is shown in Figure 7.

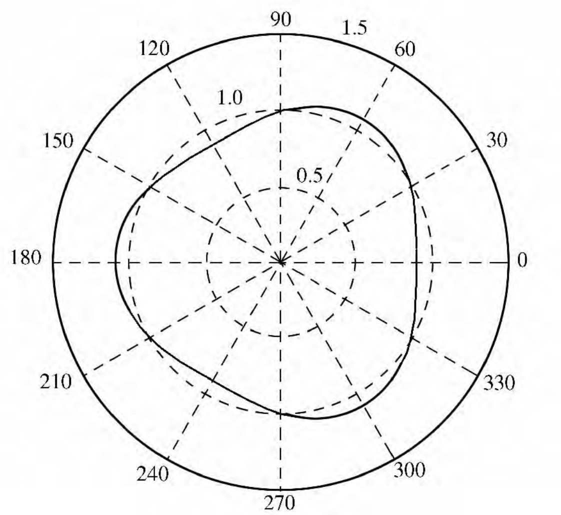

Figure 7 The trajectory of workpiece rotation when the swinging frequency = 3 times rotational frequency

In Figure 7, the cross section contour is a closed curve but radius is regularly changed. Based on the ideal circle which radius is 1, we can find that the cross-sectional profile is filled with several tiny waveforms.

If $n = 5$, the cross section graphed by MATLAB is shown in Figure 8.

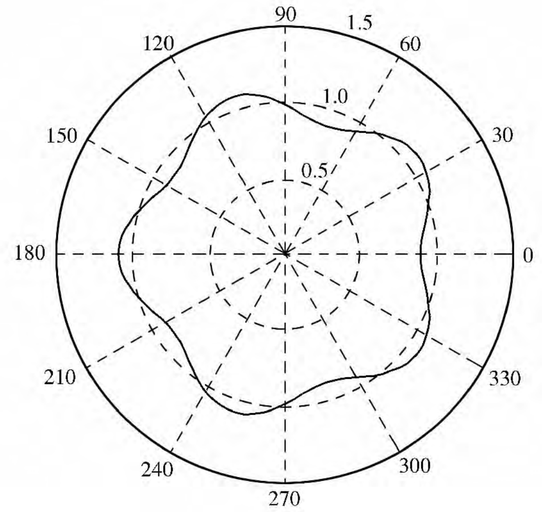

Figure 8 The trajectory of workpiece rotation when the swinging frequency = 5 times rotational frequency

From Figure 7, Figure 8 and some other cross-sectional profile graphics with different swinging frequencies, we know the waveform on the workpiece cross-section contour is increased with the increase of swinging frequency. And the number of the crest and trough is equal to the value of n. Because of the center line of the wave overlapping with the ideal workpiece contour, we can say that the ideal diameter of the workpiece is the actual diameter of the workpiece, and it is not associated with the swinging frequency.

4 Calculation and analysis of taper

When it comes to the cylindrical workpieces, its taper is equal to (larger diameter D minus the path d) over axial length. From the previous analysis we know that, when the spindle swinging frequency is low, the diameter of the workpiece is determined by the least-square circle of cross-sectional profile (the envelope curve); when spindle swing frequency is high, the diameter of the workpiece is determined by the ideal circle of cross-sectional profile.

Let´s assume that the length of workpiece machining is 100mm, the diameter is 30 mm, and the lathe spindle amplitude of angular swing is 0.01.

4.1 Taper analysis at low swing frequencies

Assuming $n = 0.1$, the workpiece cross section diameter changes at different processing length as shown in Figure 9.

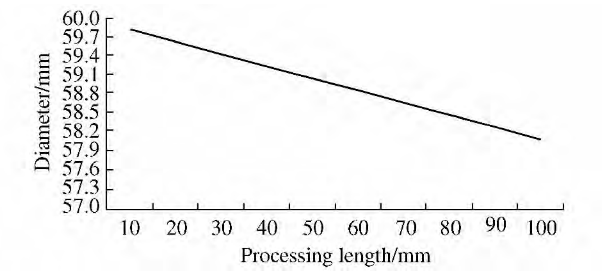

Figure 9 The cross-section ideal diameter under the different processing length

Figure 9 shows that when the swinging frequency is constant, the cross-sectional diameter of workpiece decreases linearly with the growth of the length of processing. If the spindle angular swings in the plane, the workpiece will definitely generate tapers. With the assumed conditions, the calculation of the workpiece´s taper is $(59.804-58.048)/100=0.01756$.

When the swing frequency is lower than the spindle rotation frequency, the different swinging frequencies cause the tapers changing. The relationship is shown in Table 1.

From Table 1, it can be seen that there is no linear correlation between the tapers and the swinging frequency. But when the swinging frequency is half of the rotation frequency ($n=0.5$), the workpiece taper suddenly drops to the minimum value. It indicates that when $n=0.5$, spindle swinging has a minimal impact on the workpiece taper. Therefore, in the actual design and production, if the spindle swinging cannot be avoided, we should try to make swinging frequency close to half the value of rotation frequency to minimize taper error.

Table 1 Taper of the workpiece at different low swing frequencies

Frequency doubling n	Taper/mm
0.0001	0.0198
0.001	0.0198
0.01	0.01978
0.1	0.01756
0.2	0.01792
0.3	0.01932
0.4	0.01792
0.5	0.0099
0.6	0.01792
0.7	0.01932
0.8	0.01792
0.9	0.01932

4.2 Taper analysis at high swing frequency

When the spindle swing frequency is greater than the spindle rotation frequency, referring to the conclusions of 3.2.2, at different swing frequencies, the ideal diameter and the workpiece diameter are always equal. Therefore, with the assumed conditions, no matter how n changes; the workpiece will not generate taper. But its surface will generate larger waviness, roughness and the fluctuation of the cross-sectional contours will enlarge with the length of processing, so in this case the influence of the spindle swinging mainly reflects on the significant change of the surface quality.

5 Conclusions

The spindle rotation error of machine tool significantly matters to the precision machinery design and manufacture. This paper analyzes the taper error caused by spindle swinging at different swinging frequencies and fills the blank theoretically in this field.

When the machine tool spindle occurs angular swingings, the workpiece taper changes along with the spindle swinging frequency. Specifically, when spindle swing frequency is high, the workpiece does not generate the taper, but it has great influence on surface quality; when the spindle swinging frequency is low, the workpiece generates the taper, and when the swinging frequency is half of the rotation frequency, the taper approaches its minimum. In the practical design and production, the specific association should be fully taken into account. If the machining precision cannot meet the requirements, the equipment should be check in time and eliminate the factors that cause the spindle system angular swinging.

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