DOI: 10. 13434/j. cnki. 1007-4546. 2014. 0205

Research for Non-probabilistic Structure Reliability

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Abstract: The paper discusses non-probabilistic approaches for uncertainty treatment in structure reliability analysis. Based on rough set theory, the uncertain parameters of structures are expressed by rough variables, the structure reliability index is computed by rough function and metric. This new methodology for structural reliability is proved to be valid and efficient using theory analysis and examples of practical application.

Key words: non-probabilistic; structure reliability; rough set theory (RST); rough function

1 Introduction

A probability-based reliability assessment^[1] requires precise probabilistic characteristics of the random inputs, such as the Advanced First-Order Second Moment (AFOSM) model, the Second-Order Second Moment (SOSM) model, the High-Order Moment model, Optimization Method, the Response Surface Method, Monte-Carlo Simulation and the Stochastic Finite Element Method. Reliability analysis begins by identifying the sources of uncertainty-loads, failure conditions, material or geometrical properties and so on. These data , however , are sometimes practically difficult to obtain, especially when only a limited number of input samples are available or the uncertainties are inherently non-probabilistic. Some attempts have been made by both the engineering and the applied mathematics community for tackling this

challenging problem.

However, as revealed by Z. P. Oiu^[2], unjustified assumptions in constructing a probabilistic model for input quantities may yield misleading results in the probabilistic reliability analysis. This means that the traditional probabilistic approaches may be questionable to deal with some problems involving incomplete information or inherently non-probabilistic uncertainties. Therefore, non-probabilistic uncertainty models, such as the convex model and the fuzzy set, have been considered as beneficial supplements to the traditional probabilistic model^[34]. One important merit of the convex model is that it can not solve scale factor , but also the extreme parameter combinations are excluded. The fuzzy set needs membership function, but it depends on factors created manually. Guo Shuxiang propose definitional procedure, transformational procedure and procedure by optimization, but calculation of the workload is very large, Monotonic variable is not easy to be determined; Boundary is often difficult to obtain; Affine arithmetic needs complicated calculation.

Received 28 March 2014

The paper is supported by National Natural Science Foundation of China under Grant No. 61373112, Special Project of Scientific Research of Education Department of Shaanxi Provincial Government No. 11JK0967

Rough set philosophy^[5] is based on the assumption that , in contrast to the classical set theory , we have some additional information (knowledge , data) about elements of a set. Elementary concepts can be combined into compound concepts. Any union of elementary sets is called a crisp set , and other sets are referred to as rough (vague , imprecise) .

Taking the indiscernibility relation defined on real number set as the basic starting point , the concepts of lower rough (discrete) and upper rough (discrete) representations are defined in rough function model which is based on rough set theory. A series of discrete properties of the lower and upper representations corresponding to real functions are discussed to investigate the relationship between real and discrete functions, especially how does the discrimination of real line influence basic properties of real functions, etc. Rough function model has some overlaps with mathematical such as nonstandard analysis^[6], finite analysis^[7], infinitesimal analysis^[8] and so on. Moreover, rough function model can be viewed as a generalization of qualitative reasoning^[9], in other words, threevalued qualitative derivatives are replaced by more general concept of multi-valued qualitative derivations, so that more generalized concepts in various levels can be described. Furthermore, on the basis of fault allowed theory and rough sets theory, if the rough functions of some real function is gained in advance, then the changing states of this real function can be depicted by analyzing the properties of rough functions.

The main advantage of rough set theory is that it does

not need any preliminary or additional information about data like probability in statistics, or basic probability assignment in Dumpster-Shafer theory and grade of membership or the value of possibility in fuzzy set theory.

In this paper we develop a rigorous quantitative nonprobabilistic measure of reliability , based on RST. It can tolerate a large amount of uncertainty before failure occur.

2 Model of non-probabilistic structural reliability

The uncertainty in the specification of these design– base inputs is expressed with RST. The uncertainty design-based inputs are denoted X_i ($i \in n$). The ob– jective function of the non-probabilistic reliability is defined as

$$M = g(X_i) = g(X_1 X_2 , \dots X_n)$$
(1)

Where the vector $X_i = \{X_1, X_2, \dots, X_n\}$ is a collection of basic structure-related interval variables , such as Young's modulus and the ultimate tensile strength. The failure criterion of a structure is determined by setting $g(X_i) = 0$. Moreover, $g(X_i) > 0$ and $g(X_i)$ <0 denote the safe region and the failure region for analyzing the working condition of a structure respectively. Therefore , the surface satisfying $g(X_i) = 0$ is usually called the failure surface and $g(X_i)$ is a continuous function of the interval variables X_i . The structure reliability index is denoted by β .

Doing this for each element of X_i generates a new set , which is an expanded (or contracted) version of RST.

$$apr(X) = \bigcup \{ x \in U : \mu_X = 1 \}$$
 (2)

$$\overline{apr}(X) = \bigcup \{ x \in U : \mu_X > 0 \}$$
(3)

The interval model defines the variation range of a structural parameter as an interval set bounded by its lower and upper bound. Let \overline{X} and X denote the

 (i_1)

lower bound and the upper bound of an uncertain parameter X , respectively.

$$\underline{X_i} = \left\{ \left[\frac{a_i + b_i}{2} \right] \right\}$$
(4)

$$X_i = \begin{bmatrix} a_i & b_i \end{bmatrix}$$
(5)

Using the notation , all possible values of X can thus be modeled by an interval set , which is expressed by $[\overline{X}, \underline{X}]$. However , by using it , the result is ex-

pand , and the robust reliability criterion is not true for multidimensional structures.

In this study, the new rough function model are developed to address the aforesaid concerns and they are incorporated into RST to produce a more efficient computational scheme for determination of the nonprobabilistic reliability of a structure.

The processes of expansion and translation can be combined , as indicated in the following definition^[5].

Definition 2. 1. Let $[n] = \{0, 1, 2, \dots, n\}$ be a finite set of integers, R be the set of real numbers. If the strictly monotonic function d: $[n] \rightarrow R$ satisfies: $\forall i$, $j \in [n]$, i < j, implies d(i) < d(j), then d is referred to as a scale.

Definition 2.2. For d_i : $[n_i] \rightarrow R$, $i \in \{0, 1, \dots, n\}$

and $e [m] \rightarrow R$ are two scales $f : R_{n_1} \times R_{n_2} \times \cdots \times R_{n_n}$ $\rightarrow R_m$ is a *n*-real function.

Include that:

$$f_* (i_1 \ i_2 \ i_3 \ \cdots \ i_n) = e_* (f(x_{i_1} \ x_{i_2} \ \cdots \ x_{i_n})) (6)$$

$$f^* (i_1 \ i_2 \ i_3 \ \cdots \ i_n) = e^* (f(x_{i_1} \ x_{i_2} \ \cdots \ x_{i_n})) (7)$$

$$i_2 \ i_3 \ \cdots \ i_n) = D_{\tau} (x_{i_1} \ x_{i_2} \ \cdots \ x_{i_n}) \ \pi = 1 \ 2 \ \cdots 2$$

$$D_{1}(x_{i_{1}} x_{i_{2}} ; \cdots x_{i_{n}}) =$$

$$(d_{1*}(x_{i_{1}}) d_{2*}(x_{i_{2}}) d_{3*}(x_{i_{3}}) ; \cdots d_{n*}(x_{i_{n}}))$$

$$D_{2}(x_{i_{1}} x_{i_{2}} ; \cdots x_{i_{n}}) =$$

$$(d_{1*}(x_{i_{1}}) d_{2}^{*}(x_{i_{2}}) d_{3*}(x_{i_{3}}) ; \cdots d_{n*}(x_{i_{n}}))$$

$$D_{3}(x_{i_{1}} x_{i_{2}} ; \cdots x_{i_{n}}) =$$

$$(d_{1*}(x_{i_{1}}) d_{2*}(x_{i_{2}}) d_{3}^{*}(x_{i_{3}}) ; \cdots d_{n*}(x_{i_{n}}))$$
...

$$D_{2^{n}}(x_{i_{1}} \ x_{i_{2}} \ ; \cdots \ x_{i_{n}}) = (d_{1}^{*}(x_{i_{1}}) \ d_{2}^{*}(x_{i_{2}}) \ d_{3}^{*}(x_{i_{3}}) \ ; \cdots d_{n}^{*}(x_{i_{n}}))$$

$$f_{*} = \min\{e_{*}(f(D_{\tau}(x_{i_{1}} \ x_{i_{2}} \ ; \cdots \ x_{i_{n}})))\} (8)$$

$$f^{*} = \max\{e^{*}(f(D_{\tau}(x_{i_{1}} \ x_{i_{2}} \ ; \cdots \ x_{i_{n}})))\} (9)$$

Definition 2.3. Forf $[n] \rightarrow [m]$ is a rough function, if $\forall i \ j \in [n]$, while $|i - j| \leq 1$, have $|f(i) - f(j)| \leq 1$, hence f is rough continuous function.

According to the definition of rough set model and the generalized non-probabilistic reliability index^[5], we can derive the relative reliability β as follows:

$$\beta = \frac{g_*(X) + g^*(X)}{g^*(X) - g_*(X)} = \frac{e_*(g(X_{i_1} | X_{i_2} ; \cdots ; X_{i_n}) + e^*(g(X_{i_1} | X_{i_2} ; \cdots ; X_{i_n}))}{e^*(g(X_{i_1} | X_{i_2} ; \cdots ; X_{i_n}) - e_*(g(X_{i_1} | X_{i_2} ; \cdots ; X_{i_n}))} = \frac{\min\{e_*(g(D_{\tau}(X_{i_1} | X_{i_2} ; \cdots ; X_{i_n})))\} + \max\{e^*(g(D_{\tau}(X_{i_1} | X_{i_2} ; \cdots ; X_{i_n})))\}}{\max\{e^*(g(D_{\tau}(X_{i_1} | X_{i_2} ; \cdots ; X_{i_n})))\} - \min\{e_*(g(D_{\tau}(X_{i_1} | X_{i_2} ; \cdots ; X_{i_n})))\}}$$
(10)

Obviously , the greater the non-probabilistic reliability index (beita) is , the greater extent of parameter variation the structure will allow for. Particularly , $\beta = 1$ means that the structure is critical for the reference parameter uncertainties. For $\beta > 1$, all the possible values of the uncertainties lie within the safe region and therefore the structure has a safety margin. Though it might be argued that $\beta = 1$ is sufficient for a reliability requirement if the chosen ellipsoids (or intervals) reflect well the actual variability of the structure , a greater value of β offers a specified safety margin , which is usually desirable in the practical engineering.

3 Numerical examples

Example Consider a limit state function of structure in

e(i)

the following form: $G = 543RS - 0.3H^2$, inside variables $R \in [0.572 \ 0.668]$, $S \in [2.073 \ 2.288]$, $H \in [30.54 \ 31.22]$.

Normalizing each of the interval variables by substitu-

ting $\underline{R} \in [0.62]$, $\overline{R} \in [0.572, 0.668]$, $\underline{S} \in [2.181]$, $\overline{S} \in [2.073, 2.288]$, $\underline{H} \in [30.88]$, $\overline{H} \in [30.54, 31.22]$, G is a 3-real function, $G = f(x, y, z) = 543xy - 0.3z^{2}$, $d_{1}(i) = 0.572 + 0.048i$

$$d_{2}(i) = 2.073 + 0.108i$$

$$d_{3}(i) = 30.54 + 0.34i$$

$$e(i) = 226.09 + 20.001i$$

$$x_{i} = d_{1}(i) \quad y_{i} = d_{2}(i) \quad z_{i} = d_{3}(i) \quad w_{i} =$$

In light of formula(10) using the modified computational scheme explained above:

 $\beta = 1.265$

4 Conclusions

We proposed a new non-probabilistic model of reliability analysis. The results show that the proposed scheme is capable of reducing computational complexity significantly and at the same time it can preserve the calculation accuracy. This method is considered as more reliable and efficient than the existing ones.

References

- Moens D , Vandepitte D. Recent advances in non-probabilistic approaches for non-deterministic dynamic finite element analysis [J]. Arch. Comput. Meth. Engrg , 2006 ,13: 389– 464
- [2] Qiu Z P, Mueller P C, Frommer A. The new non-probabilistic criterion of failure for dynamical systems based on convex models [J]. Math Comput Model, 2004 A0

- [3] Lin P L, Kiureghian A D. Optimization algorithms for structural reliability [J]. Structural Safety ,1991 ,120(9):161-177
- [4] Moller B , Beer M. Engineering computation under uncertainty-capabilities of non-traditional models [J]. Comput. Struct , 2007 86: 1024–1041
- [5] Pawlak Z. Rough sets rough function and rough calculus [C]//Rough-Fuzzy Hybridization: A New Trend in Decision-Making. SPringer-Verlag ,1999
- [6] Mycielski J. Analysis without actual infinity [J]. Journal of Symbolic Logic , 1981 46: 625–33
- [7] Ruokolainen J. Constructive nonstandard analysis without actual infinity [M]. Porthania: Helsinki, 2004
- [8] Chuaqui R , Suppes P. Free-variable axiomatic foundations of infinitesimal analysis: a fragment with finitary consistency proof [J]. Joumal of Symbolic Logic , 1995 60: 122-159
- [9] Werthner H. Qualitative reasoning-modeling and the generation of behavior [M]. Singapore: SPringer-Verlag 1994

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