A Review of Structural Durability and Probabilistic Damage Tolerance

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Abstract: The main purpose of this paper is providing a reference for further research. According to the papers and reports on structural durability and probabilistic damage tolerance, the present paper summarized the progress of the theoretical considerations and engineering application. Several models used in structural durability and probabilistic damage tolerance are reviewed. The characteristics and problems of these methods are analyzed. A new kind of combined analysis model on structural durability and damage tolerance are also introduced. New progress of analysis theory and numerical methods on structural reliability are discussed, such as the response surface method and numerical method combining neural networks and Monte Carlo simulation. The analysis shows that these methods can improve computational efficiency significantly and maintain high computational accuracy. Finally, some prospects of the key research directions are discussed.

Key words: structural durability; probabilistic damage tolerance; numerical method

1 Introduction

Structural durability is a measure of service life under the prescribed conditions of use and maintenance, and a form of expression of the structural reliability. The main purpose of structural durability analysis is to predict economic life. The main purpose of damage tolerance analysis is to acquire the residual strength and the crack propagation life of the structure, to ensure structures meeting the residual strength requirement, and to determine the crack inspection cycle. The structural durability analysis is usually used in predicting the service life, and the damage tolerance analysis to ensure the structural safety. In order to ensure that the structure meets the residual strength requirements in high reliability, and to predict the crack propagation life in specified reliability requirements, the probabilistic damage tolerance analysis methods are raised. Durability analysis is primarily concerned with the process of crack initiation, and probabilistic damage tolerance analysis with crack propagation.

Received 15 April 2013

2 Methods for durability analysis

Durability analysis aims to analyze the degree of structural damage, which is usually given by crack exceedance in specified reliability requirements, and predict the economic life of the structure according to the specified allowable damage. The commonly used methods for durability can be classified into three classes: probabilistic fracture mechanics method (PFMA), deterministic crack growth method (DCGA) and crack initiation method (CIA).

Durability analysis should obtain the structural details of the original manufacturer state and the original fatigue quality (IFQ) of a structure. That IFQ is usually represented with time to crack initiation (TTCI) and equivalent initial flaw size (EIFS). The TTCI is time for a crack to reach the reference crack size under a given stress spectrum. The TTCI distribution can be obtained by fitting the standard distribution model parameters using experimental sample data. Commonly used TTCI distribution models include the two-parameter Weibull model, three-parameter

Weibull model and lognormal model. The EIFS distribution uses the compatible functions corresponding with TTIC models, its parameters usually obtained by deducing from TTIC [1] .

The probabilistic fracture mechanics method [2] is a durability analysis method proposed in the seventies of the last century; this method directly studies the variation of the crack distribution over time and obtains the relationship between the degree of damage and the service time. Then the crack exceeding probability criteria or ratio of repair and replacement costs criteria is employed to ensure that the structure meets durability design requirements.

The deterministic crack growth method confirms whether the structure meets durability requirements by verifying typical details of the most serious stress zone exceeds economic repair limit in 2 times service life and the details of the "worst" of the stress zone in service life. This method is suitable for the durability analysis of preliminary design and detailed preliminary design stage.

The crack initiation method was developed in 1992 by Wen-Ting Liu [3] . This method connected the reliability of safe life under load spectrum with the crack exceeding probability on the basis of P-S-N curves for structure members′ crack initiation. The structure damage degree was evaluated by establishing the $P(i, t)$ curve, then the structure economic life is predicted. The generic EIFS models are not required in this method.

In 2006 Yang Mou-Cun [4] assumed that the structural details of the initial fatigue quality are not related to the shape, process parameters, or assembly state, but only to material defects. Based on this assumption, original material fatigue quality (MIFQ) was defined. The relationship between the stress serious coefficients and crack propagation rate was established to predict the crack propagation life of different connecting members. Based on this theory, Yang proposed a new method for durability analysis which does not require a large amount of test data, and can effectively reduce the cost and time.

3 Methods for probabilistic damage tolerance analysis

Considering the uncertainty of structural crack propagation and the critical crack size in damage tolerance analysis, the main content of probabilistic damage tolerance is linking residual strength and crack propagation life to reliability. Probabilistic damage tolerance analysis aims to ensure that the structure meets the residual strength requirements under high reliability. The probabilistic damage tolerance analysis includes stochastic crack grow analysis and reliability analysis.

3.1 Stochastic crack growth model

Taking randomness of many factors that affect the fatigue crack growth into account, scholars proposed many random crack propagation models. The typical method is randomization of differential equations.

The stochastic crack growth model proposed by Yang [5] is the most widely used model and it is expressed as follows

$$\mathrm{d}a/\mathrm{d}t = X(t)\, L(\Delta K, K_{\max}, R, S, a) \quad (1)$$

Where $X(t)$ is a non-negative stationary random process representing the uncertainness in crack growth; $L(\Delta K, K_{\max}, R, S, a)$ is a non-negative function; ΔK is the stress intensity range; $K_{\max}$ is maximum value of stress intensity; S is the load spectrum; a is the crack length and t is the service life in the load spectrum. Based on this model, Yang proposed a lognormal stationary stochastic crack growth model and obtained a first order approximate calculation method for the case that initial crack length was a constant.

Many scholars made improvement and promotion for the above model. Jin Xing [6] improved Yang′s model, taking the randomness of load and initial crack length into account, and broaden the scope. Also, he proposed a practical engineering method using the first order approximate method. Liu Wen-Ting[7] proposed a normal random crack growth model and gave a parameter estimation using the maximum likelihood method. This model is applicable to the case of the smaller crack propagation.

The Markov chain model[8,9] discretes time with so-called duty cycles, and the damage states are labelled by state $j = 0, 1, 2, \cdots, b$ where state 0 is defined as the initial damage which cannot be related to a measurable crack size. In this way, the process of damage accumulation can be described using a discrete-time, discrete-state Markov chain. The probability of damage state at any moment is represented by a vector. The main advantages of this method are that the crack size distribution within any duty cycle can be obtained easily, and it gets easier to determine the detection means and the first detection time.

3.2 Reliability model of residual strength

The reliability model based on residual strength was first proposed by Kececioglu[10]. The criterion of structure safety for this model is: whether the crack tip stress intensity factor exceeds the fracture toughness after a given time. Considering the impact of random factors, the analysis model of reliability can be proposed. The most commonly used model is the $K \sim K_c$ interference model. The probability of failure of the structure is

$$P_F = P(K > K_c) = 1 - \int_0^\infty F_K(K_c) f_{K_c}(K_c)\, dK_c \tag{2}$$

Correspondingly, the reliability is

$$R = \int_0^\infty F_K(K_c) f_{K_c}(K_c)\, dK_c \tag{3}$$

Where $F_K(K)$ is the probability distribution function of the stress intensity factor at time t and $f_{K_c}(K)$ is the probability density function of the fracture toughness.

The distribution function of the stress intensity factor can not obtained from experimental data directly, so a $a \sim a_c$ interference model is raised. The probability of failure of the structure is

$$P_F = P(a > a_c) = 1 - \int_0^\infty F_a(a_c) f_{a_c}(a_c)\, da_c \tag{4}$$

Correspondingly, the reliability is

$$R = \int_0^\infty F_a(a_c) f_{a_c}(a_c)\, da_c$$

Where $F_a(a)$ is the distribution function of the crack size at time t and $f_{a_c}(a)$ is the probability density function of the critical crack size.

Many scholars[11~13] studied the above model, made improvements and promotions, and proposed many practical engineering methods.

3.3 Reliability model of crack propagation life

This model is based on the criteria of failure life, which means: for a given design life t, the criterion for determining structural failure is whether t is longer than (or equal to) service life t_c (life for crack grows to the critical state). The reliability of the structure is constructed as

$$R(t) = 1 - P(t \geqslant t_c) = 1 - F_c(t_c)$$

Where $F_c(t_c)$ is the probability distribution function of the service life.

Liu Wen-Ting[7] proposed a method to obtain the probability distribution function of the service life using (a, t) data sets of experiment or simulation pieces. Luo Yi[14] proposed a new crack propagation life-fracture toughness model based on the crack propagation life model. Luo's model considers material fracture toughness as a random variable, then calculates the reliability from the probability distribution functions of fracture toughness K_{IC} and crack propagation life for a given K.

In fact, the reliability model of residual strength has the theoretical consistency with the crack propagation life model. In practical application, data requested by these two models is not the same: the model of propagation life requests the probability distribution or conditional probability distribution of crack propagation life in certain given conditions (such as a given crack size, load, or initial crack size, etc.); the reliability model of residual strength requests the probability distribution crack stress intensity factor or crack size in a given life. For now, the distribution of the stress intensity factor or crack size is inconclusive and it is more difficult to get its distribution directly from the perspective of an engineering test. Meanwhile, the distribution of crack propagation life has been concluded, and its data can be directly obtained through the test under certain controlled conditions, and therefore is more convenient in practical use.

Commonly used methods of durability and damage tolerance analysis can not do a comprehensive analysis for structural life and safety. In order to meet the requirements of comprehensive optimization in structural design, combined models for durability and damage tolerance analysis have been proposed. Chen Bo [15] proposed a combined model of flight structural durability and damage tolerance analysis by studying the total process of crack initiation and crack growth. It is based on the existing durability analysis method PFMA and the probabilistic damage tolerance analysis model, so it can be used to analyze the flight structural performance of durability and damage tolerance at the same time. It is propitious to analysis and optimizing the design of flight structures. ZHAO Wei-tao [16] adopted the non-probability initial crack size in the damage tolerance design handbook as the threshold value of crack size, and derived the probability distribution of initial crack size in probability damage tolerance analysis by the probability of the crack size beyond the threshold value and the results of durability analysis.

3.4 Methods for structural reliability

Eurocodes [17,18] classify the methods for reliability analysis in three levels: semi-probabilistic or level 1 methods, approximate probabilistic or level 2 methods and exact probabilistic or level 3 methods. The semi-probabilistic methods have the highest computational efficiency and easily used, but they are less accurate. The approximate probabilistic such as the first order or second order reliability methods (FORM/SORM) [19~21] where the probability of failure is based on the reliability index β, have the advantage of high computational efficiency and they maintain excellent computation accuracy. The Exact probabilistic methods compute the probability of failure from the joint probability distribution of the random variables associated with the actions and resistances. The Exact probabilistic methods are able to compute the probability of failure with the desired precision, but they are not efficient when compared to level 2 methods.

Due to the complexity of the process of crack propagation, it is difficult to calculate the reliability in damage tolerance analysis directly. Many numerical methods have been proposed to solve this problem. Monte-Carlo [22,23] simulation is one of the most popular methods. It can be applied to many practical problems, allowing the direct consideration of any type of probability distribution for the random variables, is able to compute the probability of failure with the desired precision and it is easy to implement. However, the requirement of a great number of structural analyses (one for each sample of the set of random variables) makes it not efficient. To eliminate this drawback, the use of neural networks (NN) [24,25] has been proposed to approximate structural response. Once properly trained, a NN allows the determination of the structural performances with a very small number of operations and at a fraction of the cost of the corresponding structural analysis. This methodology allows the application of MCS to practical cases of great complexity where the direct use of this method would not be feasible.

The response surface method (RSM) for structural reliability analysis can be used when the performance function is unknown, in which the iterative sequence response surface method of a quadratic polynomial is often used. The hybrid simulation method (HSM) combining Monte Carlo Simulation and RSM has great advantages in efficiency and precision.

4 Conclusions

The structure durability and probabilistic damage tolerance methods are widely used in the analysis of the structures' in-service life and reliability, and many researchers pay much attention to them. In this article, we introduce the durability and probabilistic damage tolerance methods that have been in common use and the achievement of the study. However, some problems are left over and must have further study:

1) In the research process of the probabilistic damage tolerance method for the multiple cracks, many problems should be further studied and experimentally verified to include the analysis of the variables in the stochastic model of the crack propagation, the interaction of the multiple cracks and to simplify the reliability analysis models.

2) The existing analysis models are only appropriate for two-dimensional problems, and more work should be done on the probabilistic damage tolerance analysis method of three-dimensional crack propagation.

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