DOI: 10.13434/j.cnki.1007-4546.2014.0102

The *J* Integral of a Slightly Curved Elasticity-plasticity Crack when Enduring Quasi Static Loads

YANG Da-peng, CHEN Xi, ZHAO Yao, LI Tian-yun
Machanical Engineering Department, Zhengzhou Technical College, Zhengzhou 450121, P. R. China

Abstract: The *J* Integral of an elastic-plastic curved crack under quasi-static loads has been mainly worked in the paper, and the *J* Integral has been calculated as a practical application of a second order perturbation method and theorem of surname KA where the effect of quasi-static applied stresses and normal and shear stresses on the boundaries of plasticity area are synthetically taken into consideration. A regular pattern of variations of the *J* Integral of an elastic-plastic curved crack with the variations of curved crack shape parameters under different quasi-static loads has been mainly carried out. Continuity and unity of a elastic-plastic fracture and linear elastic fracture has been demonstrated from the viewpoint of the mechanical parameter of the curved crack tip *J* integral. The elastic-plastic slightly curved crack tip *J* integral has been calculated approximately, an integral loop having been reasonably selected and continuity and unity of the fracture characteristic of centre penetration straight line crack and curved crack have been demonstrated.

Key words: curved crack; quasi-static; second order perturbation solution; *J* Integral

1 Introduction

The quasi-static slightly flexed crack perturbation analysis was originally performed by Banichuk [1] 、Goldstein and Salganik [2,3]. Having obtained a simple expression of stress intensity factors by using the same method, Cotterell and Rice [4] studied the slightly flexed crack growth path of a semi-infinite crack in the infinitely extended plane. The expansion problem of a slightly flexed crack have been studied, respectively, by Karihaloo et al [5] and Sumi et al [6,7] using the first and second order perturbation methods. Yoichi Sumi [8] calculated the approximation of an intensity factors of elastic-plastic curved crack under an arbitrary far field boundary condition. Wu [9] 、Amestoy and Leblond [10-12] also have presented the exact asymptotic results of intensity factors of curved crack and shape parameters of a growing path.

But researches on the curved crack growth path are all in the state of linear elasticity [13-19]. Research findings concerning the *J* Integral have merely been confined to a straight line crack [18-22]. The *J* Integral of an elastic-plastic curved crack tip has not been carried out [18-22]. In this paper, the *J* Integral of an elastic-plastic curved crack with the variations of curved crack shape parameters under different quasi static loads has been studied, and *J* Integral has been calculated by the second order perturbation method.

2 Solution of the slightly curved crack tip *J* integral

2.1 The selection of *J* integral loop

Uniform distribution state of the stress σ_1 and σ_2 has been recognized [20] which are acting at the crack point

Received 25 January 2014
The paper is supported by Research the National Natural Science Foundation of China under Grant No.91016026

in wedge tape of form plasticity zone at top and bottom, moreover, Figure 1 zooms in a display further.

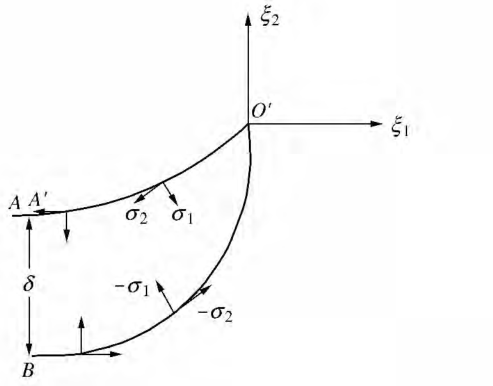

Figure 1 Positive stress σ_1 and shearing stress σ_2 acting at the crack tip wedge tape form of plasticity zone in top and bottom land

Because the value of the J integral is one constant having nothing to do with the integral loop, and it is conservative [22], therefore, analogous to the K factor within the linear elastic crack problem, the J integral reflects certain mechanics characteristics and stress strain field strength of the crack tip. There are two points A and B on the place where the curved crack tip opening displacement is δ, and they are also on top and bottom of a hypothetical curved crack, as shown in Figure 1.

The on-line of A and B should be perpendicular to an arc segment AO' and arc segment BO'; Because the curving crack curvature is small, for convenience in this research, another point A' can be got beside point A to make the on-line of A and B run parallel with the ξ_2, and the distance between two points approximates δ, then take an arc combinatorial segment $BO'A'$ for integral loop Γ, then we have:

$$J = \int_\Gamma \left(W \mathrm{d}\xi_2 - T_i \frac{\partial u_i}{\partial \xi_1} \mathrm{d}z \right) \quad (i = 1, 2) \qquad (1)$$

Among them, arc segment BO' and arc segment $O'A'$ are parallel, their shape parameters are α, β and γ which have been already recognized [20], W is the strain energy density $W = \int \sigma_{ij} \mathrm{d}\varepsilon_{ij}$ of one random point (x, y) on loop Γ; T_i is the stress components of one random point (x, y) on loop Γ along shaft ξ_1 and shaft ξ_2 respectively; Here the included angle is supposed to be θ which is between shaft ξ_1 and the tangential line of one random point on the boundary line AO' in the plasticity region; u_i is the displacement quantity of one random point (x, y) on loop Γ; The $\mathrm{d}z$ is the infinitesimal arc cell of loop Γ.

Because the curvatures of arc segment AO' and arc segment BO' are small, and signs of $\mathrm{d}\xi_2$ is opposite contrary at arc segment BO' and arc segment $O'A'$, the strain energy density W is a scalar quantity, hence an equation $\int_\Gamma W \mathrm{d}\xi_2 = 0$ exists, $\mathrm{d}z = \mathrm{d}\xi_1$; The T_i acts on the loop path, in BO' there exists $T_2 = -(\sigma_1 \cos\theta + \sigma_2 \sin\theta)$, in $O'A$ there exists $T_2 = \sigma_1 \cos\theta + \sigma_2 \sin\theta$; These two $T_2 s$ on these two arc segments are both perpendicular to ξ_1 shaft, directions reversal; Displacement quantisty ui of the path corresponding to operation force T_2 only have the v of ξ_2 direction. In the integral item $\int_\Gamma -T_2 \frac{\partial v}{\partial \xi_1} \mathrm{d}z$, the $\mathrm{d}z$ equals $\mathrm{d}\xi_1$ approximately, T_i acting on the path still exists $T_1 = -(\sigma_2 \cos\theta - \sigma_1 \sin\theta)$ in the BO', in the $O'A$ there is also $T_1 = \sigma_2 \cos\theta - \sigma_1 \sin\theta$; These two $T_1 s$ on these two arc segments are both perpendicular to ξ_2 shaft, directions reversal; Displacement quantisty of the path ui corresponding to operation force T_1 only have the u of ξ_1 direction. In the integral item $\int_\Gamma -T_1 \frac{\partial u}{\partial \xi_1} \mathrm{d}z$, the $\mathrm{d}z$ equals $\mathrm{d}\xi_2$ approximately, $\int_{BO'} -T_1 \frac{\partial u}{\partial \xi_1} \mathrm{d}z$ equals $\int_{O'A'} -T_1 \frac{\partial u}{\partial \xi_1} \mathrm{d}z$ with integral absolute value among them, sign reversal, therefore the value of this item is zero.

As shown in Figure 1, take two segmental arc combinations as an integral loop, the direction is counterclockwise.

2.2 Calculation of the J integral

Because the stress distribution state of the slightly curved crack tip plasticity zone boundaries has been already known, and because there exists $T_2 = -(\sigma_1\cos\theta+\sigma_2\sin\theta)$ on arc segment BO'

$$\tan\theta = \alpha + \frac{3}{2}\beta x_1^{\frac{1}{2}} + 2\gamma x_1 \quad (0 \leqslant x_1 \leqslant c) \quad (2)$$

$$\cos\theta = \frac{1}{\sqrt{1+\left(\alpha + \frac{3}{2}\beta x_1^{\frac{1}{2}} + 2\gamma x_1\right)^2}} \quad (3)$$

$$\sin\theta = \frac{\alpha + \frac{3}{2}\beta x_1^{\frac{1}{2}} + 2\gamma x_1}{\sqrt{1+\left(\alpha + \frac{3}{2}\beta x_1^{\frac{1}{2}} + 2\gamma x_1\right)^2}} \quad (4)$$

therefore, on arc segment BO'

$$T_2 = \frac{t_2\sigma_s}{t_2^2+1}\frac{\left(t_2 + \alpha + \frac{3}{2}\beta x_1^{\frac{1}{2}} + 2\gamma x_1\right)}{\sqrt{1+\left(\alpha + \frac{3}{2}\beta x_1^{\frac{1}{2}} + 2\gamma x_1\right)^2}}$$

At the plasticity zone boundaries its numerical value changes with the change of corresponding dot coordinates. But because the curve degree of crack is slight, in order to facilitate the research, the stress of a point of this arc segment can be taken as an even stress field to replace approximately the stress field of the whole plasticity zone boundaries which changes by point slightly.

Now take $x_1 = a+\frac{R}{2}$, then order

$$T_2' = \frac{t_2\sigma_s}{t_2^2+1}\left[\frac{t_2 + \alpha + \frac{3}{2}\beta x_1^{\frac{1}{2}} + 2\gamma x_1}{\sqrt{1+\left(\alpha + \frac{3}{2}\beta x_1^{\frac{1}{2}} + 2\gamma x_1\right)^2}}\right]_{x_1 = a+\frac{R}{2}}$$

thereupon, according to the above-mentioned hypothesis, substitute all above-mentioned equations into Eq (1), the numerical value of elastoplasticity curved crack tip J integral can be got:

$$\begin{aligned} J &= \int_\Gamma W\mathrm{d}\xi_2 - T_i\frac{\partial u_i}{\partial \xi_1}\mathrm{d}z = \\ &- \int_{BO'} T_2\frac{\partial u_2}{\partial \xi_1}\mathrm{d}\xi_1 - \int_{O'A'} T_2\frac{\partial u_2}{\partial \xi_1}\mathrm{d}\xi_1 = \\ &- T_2'\int_{BO'}\frac{\partial u_2}{\partial \xi_1}\mathrm{d}\xi_1 - T_2'\int_{O'A'}\frac{\partial u_2}{\partial \xi_1}\mathrm{d}\xi_1 = \\ &\frac{t_2\sigma_s}{t_2^2+1}\left[\frac{t_2 + \alpha + \frac{3}{2}\beta x_1^{\frac{1}{2}} + 2\gamma x_1}{\sqrt{1+\left(\alpha + \frac{3}{2}\beta x_1^{\frac{1}{2}} + 2\gamma x_1\right)^2}}\right]_{x_1 = a+\frac{R}{2}} \times \\ &(v_B - v_{O'} + v_{A'} - v_{O'}) \end{aligned} \quad (5)$$

Integrate Figure 1 with the existing results [20], there are some mathematical expressions in Eq (5) as follows:

$$\begin{cases} v_A = \lambda(a+R) - \lambda(a) = \\ \alpha(a+R) + \beta(a+R)^{\frac{3}{2}} + \\ \gamma(a+R)^2 - (\alpha a + \beta a^{\frac{3}{2}} + \gamma a^2) \\ v_B = v_A + \delta \\ v_{O'} = 0 \end{cases} \quad (6)$$

Substitute Eq (6) into Eq (5), the approximate numerical value of the J integral can be got as follows:

$$\begin{aligned} J = &\frac{t_2\sigma_s}{t_2^2+1}\left[\frac{t_2 + \alpha + \frac{3}{2}\beta x_1^{\frac{1}{2}} + 2\gamma x_1}{\sqrt{1+\left(\alpha + \frac{3}{2}\beta x_1^{\frac{1}{2}} + 2\gamma x_1\right)^2}}\right]_{x_1 = a+\frac{R}{2}} \\ &\{2\alpha R + 2\beta[(a+R)^{\frac{3}{2}} - a^{\frac{3}{2}}] + \\ &2\gamma[(a+R)^2 - a^2] + \delta\} \end{aligned} \quad (7)$$

Because the elastic-plastic curved crack shape parameters α, β and γ, the crack tip plasticity zone projection size R on extension line of straight line portion and the numerical value of crack tip opening displacement CTOD have already been all recognized [20, 21], so these numerical values already known can be utilized to combine Eq (7), curved crack tip J integral value can be figured out.

It can be known [20] that when the elastic-plastic curved crack shape parameters α, β and γ tend to zero, t_2 tends to infinity; again from the Eq (7), it can be known that the curved crack tip J integral converges at the central penetration crack tip J integral $\sigma_s\delta$. This shows that central penetration crack and curved crack have continuity and unity in fracture characteristics.

Several academic calculating curves have been drawn which describe variations of the curved crack tip J integral with variations of curved crack shape parameters under different loads. The variation rule of the curved crack tip J integral is very similar to that of curved crack tip plastic zone projection length and opening displacement [20-21]. See Figure 2.

It can be observed from Figure 2, the curved crack tip J integral increases with the increasing of loads, and the extent of the increasing is very large when a straight line portion of the crack is constant. When external loads and the straight line portion of crack are constant, the curved crack tip J integral decrcases with the increasing of crack curve angle, but the rate of decrease is very slow. When curved crack shape parameters accord with $2.5° \leqslant \alpha \leqslant 3°$, decreasing rate is still slower.

It can be observed from Figure 2 the longer the straight line portion of crack, the longer the curved crack tip J integral.

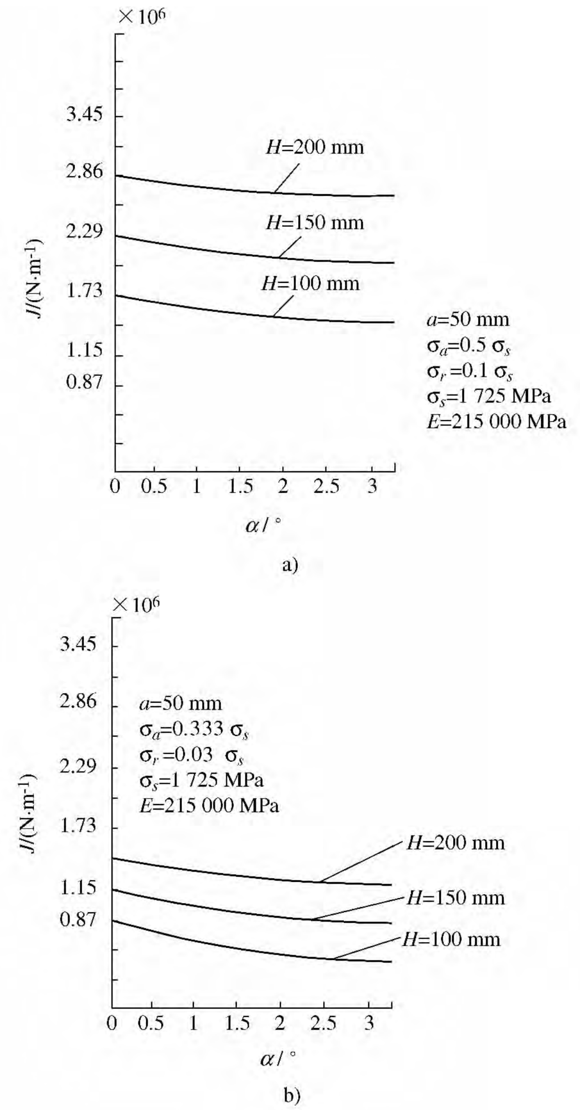

Figure 2 Academic calculating curves which describe variations of the curved crack tip J integral with variations of curved crack shape parameters under one kind of load

4 Conclusions

1) The curved crack tip J integral is increasing with the increase of loads, and the extent of increasing is very large when a straight line portion of the crack is constant;

2) When external loads and straight line portion of the crack are constant, the curved crack tip J integral decrease with the increasing of crack curve angle, but

decreasing rate is very slow; When the curved crack shape parameter α is close to or approximates 3°, decreasing rate is still slow;

3) The longer the straight line portion of crack, the longer the curved crack tip J integral.

References

[1] Banichuk N V. Determination of the form of a curvilinear crack by small parameter technique [J]. Izv, An SSSR MTT, 1970,(7-2):130-137

[2] Goldstein R V, Salganik R L. Plane problem of curvilinear cracks in an elastic solid [J]. Izv, An SSSR MTT, 1970,(7-3):69-82

[3] Goldstein R V, Salganik R L. Brittle fracture of solids with arbitrary cracks [J]. Int.J.Fracture, 1974,(10):507-523

[4] Cotterell B, Rice J R. Slightly curved or kinked cracks [J]. Int. J. Fracture, 1980,(16):155~169

[5] Karihaloo B L, Keer L M, Nemat-Nasser S, *et al*. Approximate description of crack Kinking and curving [J]. J. Appl. Mech., 1981,(48):515~519

[6] Sumi Y, Nemat-Nasser S, Keer L M. On crack branching and curving in a finite body [J]. Int. J. Fracture, 1983,(21):67-79

[7] Sumi Y. A note on the first order perturbation solution of a straight crack with slightly branched and curved extension under a general geometric and loading condition [J]. Engng Fracture Mech, 1986,(24):479~481

[8] Member Y S. A second order perturbation solution of a Non-Collinear crack and its application to crack path prediction of brittle fracture in weldment [J]. J. S. N. A. Japan, 1989,(165)

[9] Wu C H. Explicit asymptotic solution for the maximum-energy-release-rate problem [J]. Int.J. Solids Structures, 1979,(15):561-566

[10] Amestoy M, Leblond J B. On the criterion giving the direction of propagation of cracks in the Griffith theory [J]. Comptes Rendus, 301, 1985,(2):969-972

[11] Muskhelishvili N I. Singular integral equations [M]. Groningen, Noordhoff, 1953

[12] Bueckner H F. Field singularities and related integral representations in methods of analysis and solutions of crack problems [M]. Leyden, Noordhoff, 1973

[13] Stochastic model of crack paths in composites based on the Langevin equation [J]. Coon Journal of Materials Science, 2007,42(1):66-71

[14] Richard H A, Fulland M, Sander M. Theoretical crack path prediction [J]. Fatigue and Fracture of Engineering Materials and Structures, 2005,28(1-2): 3~12

[15] Glodez S, Potocnik R. Flasker J, *et al*. Numerical modelling of crack path in the lubricated rolling-sliding contact problems [J]. Engineering Fracture Mechanics, 2008, 75(3/4):880-891

[16] Roberto C, Buchholz, Friedrich G. Comparison of DBEM and FEM crack path predictions with experimental findings for a SEN-speclmen under anti-plane shear loading [J]. Key Engineering Materials, 2007, (348/349):129-132

[17] John A N. Simulation of crack growth in duc-

tile materials [J]. Engineering Fracture Mechanics, 2005, 72(6): 961~979

[18] David T, Pietro C, Nicola P. The fracture mechanics of finite crack extension [J]. Engineering Fracture Mechanics, 2005, 72(7): 1021~1038

[19] Zacharopoulos D A. Stability analysis of crack path using the strain energy density theory [J]. Theoretical and Applied Fracture Mechanics, 2004, 41(1-3): 327~337

[20] Yang D P, Zhao Y, Bai L. Research on plasticity area of slightly curved extension elasticity-plasticity crack under quasi static loads [J]. Chinese Journal of Applied Mechanics, 2010, 27(2) (in Chinese)

[21] Yang D P, Zhao Y, Bai L. Research on CTOD of slightly curved extension elasticity-plasticity crack under quasi static loads [J]. Chinese Journal of Applied Mechanics, 2010, 27(3) (in Chinese)

[22] Ding S D, Sun l M. Fracture mechanics [M]. Machinery industrial Press, 1997 (in Chinese)

Brief Biographies

YANG Da-peng is doctor of Shipping and Ocean engineering college, Huazhong University of Science and Technology, now a associate professor in the Machine Engineering Department, Zhengzhou Technical College. His research interests include engineering framewok intensity of shipping and ocean. ydpzpysh@163.com

CHEN Xi is now a prelector in Education Science College, Henan University. Her research interests include computer education science and computer application. 875190678@qq.com

ZHAO Yao is now a professor in Shipping and Ocean engineering college, Huazhong University of Science and Technology. His research interests include engineering framewok intensity of shipping and ocean. yzhaozzz@mail.hust.edu.cn

LI Tian-yun is now a professor in Shipping and Ocean engineering college, Huazhong University of Science and Technology. His research interests include engineering framewok intensity of shipping and ocean. LTYZ801@mail.hust.edu.cn